# 4 Laws of Motion


*Formula 1 racing cars have massive tyres and over the years the size of the tyres has increased. This was done to make cars faster and safer. Cars with more than a thousand horsepower are very unstable and the races are accident prone. For safety measures, the cars have wide tyres to increase the friction and stop tyres from spinning during a quick acceleration. This phenomenon is even followed in our daily life on every step. In this chapter you will learn about laws which makes our lives easier.*

## Topic Notes

- *Newton's Laws*

# NEWTON'S LAWS 1

## TOPIC 1

## NEWTON'S FIRST LAW OF MOTION

### Aristotle's Fallacy

The Greek scientist Aristotle believed that a constant external force was required to keep a body moving in a regular manner. His theory is now outdated because he only studied one side of motion and failed to explain the other, mainly, how does a moving body come to a stop? The concept of friction, an opposing external force, was invented.

### Force

Force may be defined as an external agency which changes or tends to change the state of rest or uniform motion or the direction of the motion of a body. Force has a magnitude and direction. Therefore, force is vector quantity.

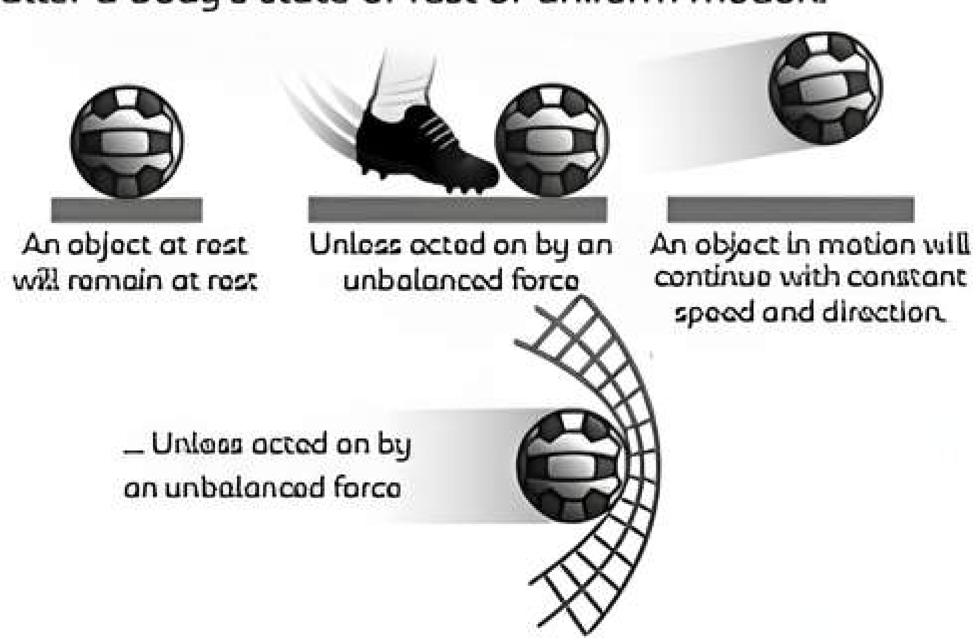
SI unit of force is Newton, which is denoted by N.

### Newton's First Law of Motion or Galileo's Law of Inertia

An object at rest remains at rest, or object in motion remains in motion at a constant speed in a straight line unless compelled by some external force to change its state.

Unless driven to alter by an external force, everybody remains in its condition of rest or uniform motion along a straight line. Inertia is the way of referring to this fundamental characteristic of the body. Newton's First Law of Motion, sometimes known as the Law of inertia. According to this law, force is defined as a factor that can modify the state of an object.

Newton's first law of motion provides a definition of force. " Force is the push or pull that alters or tends to alter a body's state of rest or uniform motion."




### Inertia

Inertia is a quality of a body that prevents it from changing its state of motion or rest. The mass of a body is used to calculate its inertia. It is directly proportional to the mass of the body.

*i.e.*, Inertia $\propto$ Mass

#### Classification of Inertia

**Inertia of rest:** The inability of a body to change its state of rest on its own. For example, passengers inside a bus or train tend to fall backward when it starts suddenly or when a blanket is beaten with a stick, dust particles fall off.

**Inertia of motion:** It is the inability of the body to change a uniform state of motion on its own. For example, when a bus or train stops suddenly, the passengers sitting inside will lean forward or when people who jump off a moving train may fall forward.

**Directional Inertia:** It is the inability of the body to change the direction of movement on its own.

**Example 1.1:** **A person who is standing freely in a bus, thrown backward, when the bus starts suddenly. Explain.**

**Ans.** When a bus suddenly starts, the force responsible for bringing the bus in motion, is also transmitted to the lower part of the body, so this part of the body comes in motion along with the bus. While the upper half of the body (say above the waist) receives no force to overcome the inertia of rest, it stays in its original position. Thus, there is a relative displacement between the two parts of the body, and it appears as if the upper part of the body has been thrown backward.

**Important**

➥ *If the motion of the bus is slow, the inertia of motion will be transmitted to the body of the person uniformly and so the entire body of the person will come in motion with the bus and the person will not experience any jerk.*

**Example 1.2:** **A person sitting in an open car moving at constant velocity throws a ball vertically up into the air. Why does the ball fall in the car ahead of the person?**

**Ans.** Because the horizontal component of velocity is the same for both car and ball, they cover equal horizontal distances in the given time interval.

**Example 1.3:** An astronaut accidentally get separated out of his small spaceship accelerating in interstellar at a constant rate of 100 m/s$^2$. Define the acceleration of the astronaut, the instant after he is outside the spaceship. (Assume that there are no nearby stars to exert gravitational force on him)

**Ans.** Since, there are no stars to exert gravitational force on the astronaut and the small spaceship exerts a negligible gravitational attraction on him, the net force acting on the astronaut, once he is out of the spaceship, is zero. By the first law of motion, the acceleration of the astronaut is zero.

## TOPIC 2

# NEWTON'S SECOND LAW OF MOTION

### Momentum

Momentum is defined as the quality of motion contained in a body. It is measured as the product of mass of the body and its velocity and has the same direction as that of the velocity. It is a vector quantity and is represented by $p$.

Momentum ($p$) = Mass ($m$) x Velocity ($v$)

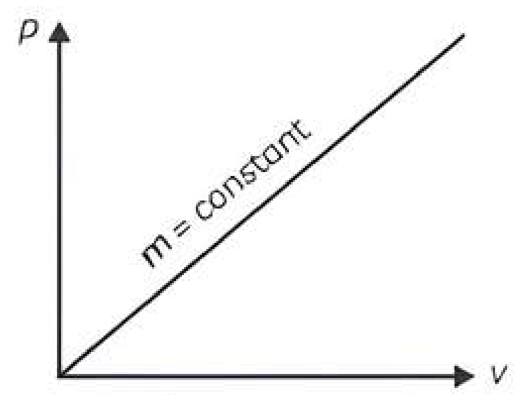
Or, $\vec{p} = m\vec{v}$

SI unit of momentum is kg. m/s

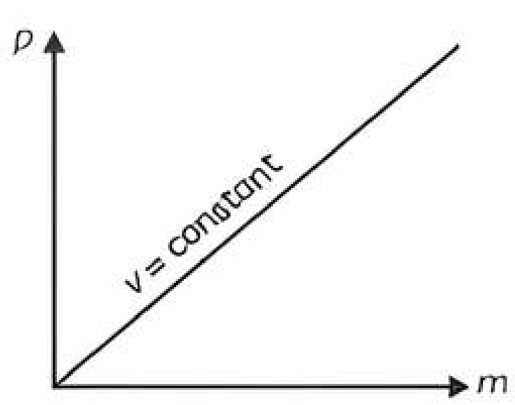
From the given equation of momentum,

we can conclude,

When $m$ is constant, $\vec{p} \propto \vec{v}$

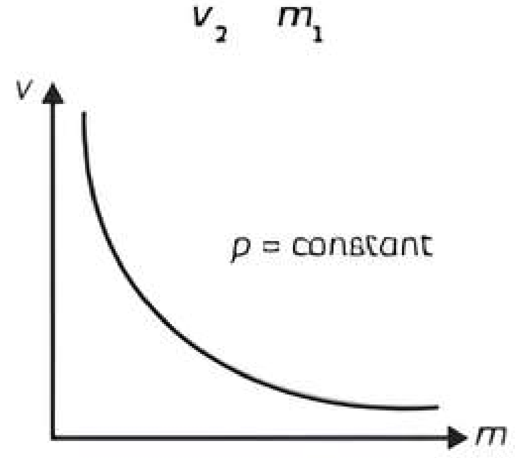



When $v$ is constant, $\vec{p} \propto m$




When two bodies of unequal masses have equal momentum then $v$ varies inversely as their masses,

$$\frac{v_1}{v_2} = \frac{m_2}{m_1}$$



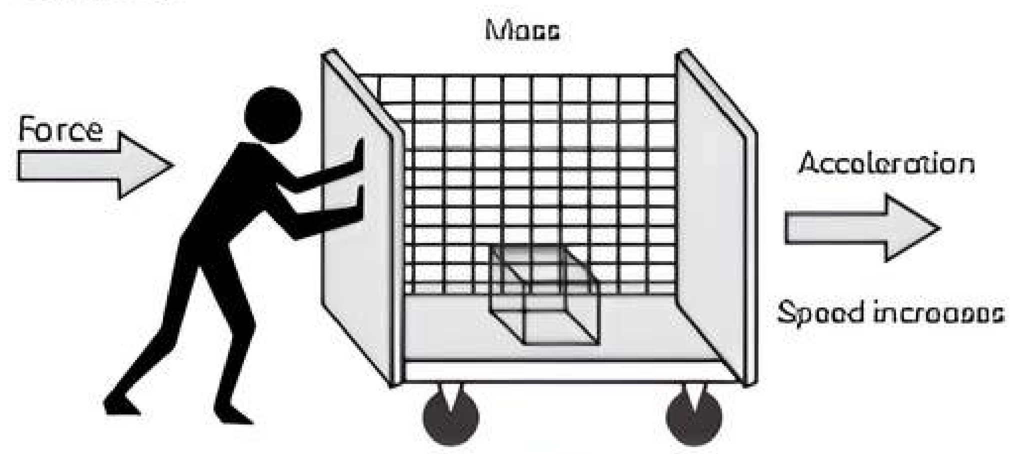

### Newton's Second Law of Motion

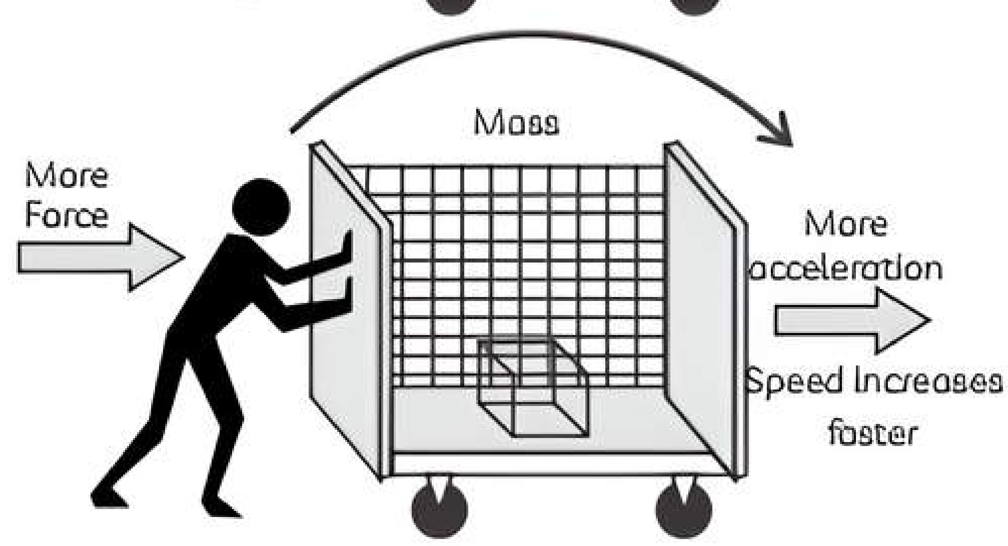
The rate of change of the momentum of an object is directly proportional to the external force applied on the body and the change takes place in the direction of the applied force.

i.e., $$\vec{F} \propto \frac{d\vec{p}}{dt}$$

or, $$\vec{F} = \frac{d\vec{p}}{dt}$$

Here, the momentum changes in the direction of the applied force. Momentum, $\vec{p} = m\vec{v}$, is a measurement of the total amount of motions, which your body contains.




**Unit force:** It is defined as the force which changes the momentum of a body by unity in unit time.

According to this, $\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v})$

$$= m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

If the mass of the system is finite and remains constant with respect to time, then $\frac{dm}{dt}=0$

and $$F=m\vec{a}=\frac{\vec{p}_2-\vec{p}_1}{t}$$

The external force acting on the object can accelerate the object by changing the magnitude of the velocity or the direction of the velocity or both.

If the force is parallel or antiparallel to the motion, it only changes the magnitude of $\vec{v}$, it does not change the direction. Therefore, the path by the body follows is a straight line.

When force is acting perpendicular to the motion of body, it only changes direction, not the magnitude of $\vec{v}$. Therefore, the path by the body follows is a circle. (Uniform circular motion).

When the force acts at an angle to the movement of the body. It changes both the magnitude and direction of $\vec{v}$. This path by the body follows may be an ellipse, a non-uniform circle, a parabola, or a hyperbola.

### Important

➥ *Second law of motion F = ma. F stands for the net force due to all material agencies external to the body. a is the effect of the force. ma should not be regarded as yet another force, besides F.*

**Example 1.4: The driver of a three-wheeler moving with a speed of 36 km/h sees a child standing in the middle of the road and brings his vehicle to rest in 4.0 s just in time to save the child. What is the average retarding force on the vehicle? The mass of the three-wheeler is 400 kg and the mass of the driver is 65 kg. [NCERT]**

**Ans.** Here, mass of three-wheeler $m_1$ = 400 kg, mass of driver = $m_2$ = 65 kg, initial speed of vehicle

$u$ = 36 km/h = 10 m/s,

final speed, $v$ = 0 m/s and $t$ = 4s.

As acceleration, $a=\frac{v-u}{t}=\frac{0-10}{4}=-2.5\ ms^{-2}$

Now, $F = (m_1 + m_2)\,a = (400 + 65) \times (-2.5)$

$= -1162.5\ N = -1.2 \times 10^3\ N.$

The –*ve* sign shows that the force is retarding force.

**Example 1.5: A bullet of mass 0.004 kg moving with a speed of 90 m/s enter a heavy wooden block and is stopped after a distance of 60 cm. Calculate the average resistive force exerted by the block on the bullet.**

**Ans.** The retardation '$a$' of the bullet (assumed constant) is given by:

$$a=\frac{-u^2}{2s}$$

$$=\frac{-90\times 90}{2\times 0.6}\ ms^{-2}=-6750\ ms^{-2}$$

The retarding force (by using 2nd law of motion) is, F

$= 0.04\ kg \times -6750\ ms^{-2}$

$= -270\ N$

The actual resistive force is negative. Therefore, retardation of the bullet may not be uniform. The answer is indicating the average resistive force.

**Example 1.6:** **Case Based:**

*Newton's Second Law of Motion says that acceleration (gaining speed) happens when a force acts on a mass (object). Riding your bicycle is a good example of this law of motion at work. Your bicycle is the mass. Your leg muscles pushing on the pedals of your bicycle is the force. When you push on the pedals, your bicycle accelerates. You are increasing the speed of the bicycle by applying force to the pedals. Another example is of a monkey climbing on a rope to reach at the top of the pillar. After reaching on the top of the grill he jumps on the building and ran away.*

(A) A monkey of mass 40 kg climbs on a rope which can stand a maximum tension of 600 N. In which of the following cases will the rope break?

The monkey:

(a) climbs up with an acceleration of 6 $ms^{-2}$.

(b) climbs down with an acceleration of 4 $ms^{-2}$.

(c) climbs up with a uniform speed of 5 $ms^{-1}$.

(d) falls down the rope nearly freely under gravity (Ignore the mass of the rope).

(B) Assertion (A): The driver in a vehicle moving with a constant speed on a straight road is a non-inertial frame of reference.

Reason (R): A reference frame in which Newton's laws of motion are applicable is non-inertial.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false

[Delhi Gov. QB 2022]

(C) A force $F=(6\hat{i}-8\hat{j}+10\hat{k})N$ produces an acceleration of $\sqrt{2}\ m/s^2$ in a body. Calculate the mass of the body.

(D) The velocity acquired by a mass *m* in travelling a distance d starting from rest under the action of a constant force is directly proportional to

(a) $\sqrt{m}$ (b) $m^o$

(c) $\frac{1}{\sqrt{m}}$ (d) M

(E) In a circus in the game of swing, the man falls on a net after leaving the swing but he is not injured, why?

**Ans. (A)** (a) *Climbs up with an acceleration of 6 $ms^{-2}$*

**Explanation:** Mass of the monkey, $m$ = 40 kg

Acceleration due to gravity,

$$g = 10\ m/s^2$$

Maximum tension that the rope can bear,

$$T_{max} = 600\ N$$

Acceleration of the monkey,

$$a = 6\ m/s^2 \text{ upward}$$

Using Newton's second law of motion, we can write the equation of motion as:

$$T = mg = ma$$
$$T = m(g + a)$$
$$= 40\ (10 + 6)$$
$$= 640\ N$$

Since, $T > T_{max}$,

the rope will break in this case.

**(B)** (c) *A is true but R is false.*

**Explanation:** Inertial frames have a constant speed since they are not accelerated.

The reference frame is an inertial frame since the driver of a car is driving at a constant pace.

Moreover, the Newtonian mechanics' (laws of motion) validity is restricted to an inertial frame.

**(C)** From Newton's second law of motion,

$$F = ma$$

Acceleration, $a=\frac{F}{m}$

or $m=\frac{F}{a}$

$$=\frac{\sqrt{6^2+8^2+10^2}}{\sqrt{2}}m$$

$$=10\ \text{kg}$$

**(D)** (c) $\frac{1}{\sqrt{m}}$

**Explanation:**

Acceleration, $a=\frac{F}{m}$

$$v^2 - 0 = 2\frac{F}{m}\times d$$

so, $v \propto \frac{1}{\sqrt{m}}$

**(E)** When the man falls on the net it is depressed where man falls on it and thus the time of contact is increased. Due to this, force of reaction on the man is reduced to a great extent. Because the increase in time reduces the impulse, which is equal to the changes in momentum ($F\Delta t = m\Delta v$). So, F is quite less and the man is not injured by the net. In fact, F pushes him up once and again he falls on the net.

## Impulse

Impulse is defined as the change in momentum. Impulse of a force, which is the product of average force during impact and the time for which the impact lasts, is measured by the total change in linear momentum produced during the impact.

$$\vec{F}=m\vec{a}$$

$$\vec{F}=m\left(\frac{\vec{v}_2-\vec{v}_1}{t_2-t_1}\right)$$

$$\vec{F}=\left(\frac{\vec{p}_2-\vec{p}_1}{t_2-t_1}\right)$$

$$\vec{F}(t_2-t_1)=\vec{p}_2-\vec{p}_1$$

$\vec{F}(t_2-t_1)$ is called the impulse of force F.

It can be written as,

$$\vec{I}=\vec{F}t$$

SI unit of impulse is Ns.

Impulse of a force acting on a body = Change in linear momentum of the body produced by the force.

Consider a force, large in magnitude, acting for a very small time interval. For example, a ball rebounding from the marble floor remains in contact with the floor for a very short time. A large force is exerted on the ball by the floor. Such a force is called an impulsive force. F is very large, t is very short, their product is finite. An impulsive force produces a finite change in momentum.

**Example 1.7:** **Hitting, kicking, catching, jumping, diving, collision etc. In all these cases an impulse act**

$$I=\int Fdt=F_{av}\Delta t=\Delta p=\text{constant}$$

**Ans.** So, time of contact $\Delta t$ is increased, average force is decreased (or diluted) and vice-versa.

In hitting or kicking a ball we decrease the time of contact so that large force acts on the ball producing greater acceleration. In catching a ball, a player by drawing his hands backwards increases the time of contact and so, lesser force acts on his hands and his hands are saved from getting hurt. In jumping on sand (or water) the time of contact is increased due to yielding of sand or water, so force is decreased and we are not injured. However, if we jump on cemented floor the motion stops in a very short interval of time resulting in a large force due to which we are seriously injured.

**Example 1.8:** **A monkey hangs from the lower end of a rope that is hung from a tree. Bananas are connected to the higher end of the rope. Will the monkey be able to eat the bananas as it climbs the rope?**

**Ans.** No. As the monkey climbs up, the bananas also move up through equal distance so that the momentum is conserved. Here the branch of the tree acts as a pulley which reverses the direction of momentum. When the monkey and the bananas both move up with the same velocity, the combined momentum remains zero

## TOPIC 3

## NEWTON'S THIRD LAW OF MOTION

According to Newton's third law of motion, for every action there is an equal and opposite reaction. When our entities A and B exert force on each other, the force by A on B (*i.e.*, force represented by $F_{AB}$ which is always equal and opposite to the force which exerts like entities B on A and the equation represents like:

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

The forces involved in any interplay among entities which are referred to as action and reaction. But we cannot say that a certain force is action and the another one is reaction.

These two terms (*i.e.*, action and reaction) always applies on different entities.

**Important**

➥ *The terms 'action' and 'reaction' in the Third Law of Motion simply stand for simultaneous mutual forces between a pair of bodies. Unlike their meaning in ordinary language, action does not precede or cause reaction. Action and reaction act on different bodies.*

**Example 1.9:** **A shell of mass 0.020 kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 ms$^{-1}$. What is the recoil speed of the gun?** **[NCERT]**

**Ans.** Given, $m$ = 0.02 kg, M = 100 kg, $v$ = 80 ms$^{-1}$, V = ?

$$V = -\frac{mv}{M}$$

$$= -\frac{0.020\ \text{kg} \times 80\ \text{ms}^{-1}}{100\ \text{kg}}$$

$$= -0.016\ \text{ms}^{-1}$$

$$= -1.6\ \text{cms}^{-1}$$

Negative sign indicates that the gun moves in a direction opposite to the direction of motion of the bullet.

## TOPIC 4

## PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

According to this principle, an isolated system preserves the vector sum of the linear momentum of all objects in the system and is unaffected by their interactions and reactions. Therefore, in a separated system; (i.e., a system without external force), mutual force between pairs of particles in the system changes the linear momentum of individual particles. However, because the forces of each other are equally opposite in each pair, changes in linear momentum in the pair are offset and the sum of linear momentum does not change. Therefore, the total linear momentum of the isolated system of interacting particles is preserved. This principle is an important result of the second and third laws of motion.

Consider an isolated system consisting of two objects A and B with an initial linear momentum a $\vec{P}_A \cdot \vec{P}_B$ . Let collide them for a short time with $\Delta t$ and separate them with a final momentum of $\vec{P}_A \cdot \vec{P}_B$ respectively. At the time of collision, if $\vec{F}_{AB}$ is force on entity A which is exerted by another entity B, and $\vec{F}_{BA}$ is force on B which is exerted by other entity A then, by applying Newton's second law represented by these equations which are given below respectively.

$\vec{F}_{AB} \times \Delta t$ = change in linear momentum of $A = \vec{p}_A - \vec{p}_B$

[for force on entity A which is exerted by another entity B]

$\vec{F}_{BA} \times \Delta t$ = change in linear momentum of $B = \vec{p}_B - \vec{p}_A$

[for force on entity B which is exerted by another entity A]

Then after applying Newton's third law of motion, (which says every action has equal and opposite reaction),

so $$\vec{F}_{AB} = -\vec{F}_{BA}$$

Therefore, from above equations,

$$\vec{p}_A - \vec{p}_A = \left(\vec{p}_B - \vec{p}_B\right)$$

or $$\vec{p}_A + \vec{p}_B = \vec{p}_A + \vec{p}_B$$

which shows that total final linear momentum of the separated system is equal to its sum of initial linear momentum. This proves the principle of conservation of linear momentum.

**Example 1.10: A weight of caravan having 1500 kg which is running with a velocity 60 km/hr on flat horizontal rails. A mass of weight 300 kg is dropped into it. Calculate the final velocity with which it runs now is:**

**(a) 25 km/h** **(b) 50 km/h**

**(c) 40 km/h** **(d) 20 km/h**

**Ans.** *(b) 50 km/h*

**Explanation:** Initially, a caravan of mass 1500 kg, which is running with velocity of 60 km/h.

So, its momentum = $1500 \times 60 \dfrac{\text{kg} \times \text{km}}{\text{h}}$

When mass of 300 kg dropped into it.

Total mass of system = 1500 + 300 = 1800 kg

Let $v$ be the velocity of the system.

Using conservation of linear momentum:

Initial Momentum = Final Momentum

$\Rightarrow$ $1500 \times 60 = 1800 \times v$

$$v = \frac{90000}{1800}$$

$$= 50 \text{ km/hr}$$

**Example 1.11: A railway car of mass 20 tones moves with an initial speed of 54 km/hr. On applying brakes, a constant negative acceleration of 0.3 m/s² is produced.**

**(A) Calculate the breaking force acting on the car.**

**(B) In what time it will stop?**

**(C) How much distance will be covered by the car before if finally stops?**

**Ans.** Given, $m$ = 20 tones = 20 × 1000 kg

$u$ = 54 km/hr = 15 m/s

$a = -0.3$ m/s², $v = 0$

(A) $F = ma$

$F = 20000 \times (-0.3)$

$F = -6000$ N

(B) $v = u + at$

$v - u = at$

$$t = \frac{v-u}{a} = \frac{0-15}{-0.3}$$

$t = 50$ s

(C) $u^2 - v^2 = 2as$

$0^2 - 15^2 = 2(-0.3)s$

$s = 375$ m

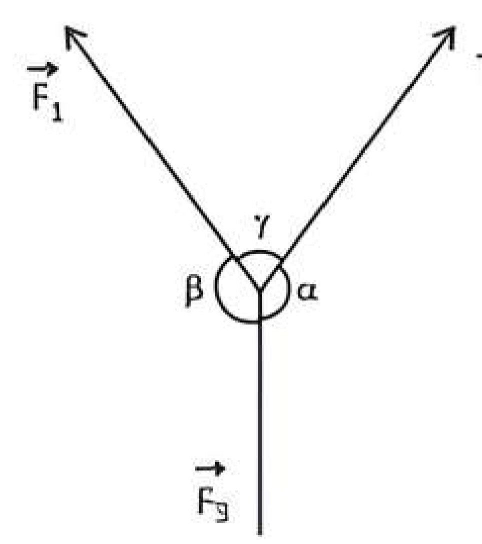
## Lami's Theorem

If many forces like, $F_1 + F_2 + F_3$ are acting concurrently on an object and the object is in the condition of equilibrium.

Then according to this theorem,

$$\frac{F_1}{\sin(\pi-\alpha)} = \frac{F_2}{\sin(\pi-\beta)} = \frac{F_3}{\sin(\pi-r)}$$

Where, $\alpha$, $\beta$, $r$ are the angles opposite to the forces $F_1$, $F_2$, $F_3$ correspondingly.




Lami's theorem is useful to solve numerical problems of electrostatics.

**Example 1.12: We have two forces having same magnitude F, which acts on an object and the magnitude of the resultant force is $\frac{F}{4}$. Find out the angle between these two forces.**

(a) $\cos^{-1}\left(-\dfrac{31}{32}\right)$ (b) $\cos^{-1}\left(-\dfrac{32}{31}\right)$

(c) $\cos^{-1}\left(-\dfrac{8}{9}\right)$ (d) $\cos^{-1}\left(-\dfrac{2}{3}\right)$

**Ans.** *(a)* $\cos^{-1}\left(-\dfrac{31}{32}\right)$

**Explanation:** Resultant of two vectors A and B, which works at an angle θ, can be given by,

$$R=\sqrt{A^2+B^2+2AB\cos\theta}$$

$$\left[\text{As } A=B=F \text{ and } R=\frac{F}{4}\right]$$

$$\left(\frac{F}{4}\right)^2 = F^2+F^2+2F^2\cos\theta$$

$$\Rightarrow \quad \left(-\frac{31}{16}\right)F^2 = 2F^2\cos\theta$$

$$\cos\theta = \left(-\frac{31}{32}\right)$$

$$\theta = \cos^{-1}\left(-\frac{31}{32}\right)$$

## TOPIC 5

# EQUILIBRIUM OF A PARTICLE

A particle which remains at rest or in uniform motion, with respect to its frame of reference, is said to be in equilibrium in that frame. According to Newton's first law of motion, if body A is at rest position will remain at rest, and a body in motion will continue in motion with constant speed in a straight line, as long as no net force acts upon the body.

This does not mean that no force acts on the particle, but that the resultant force of all the forces acting on the particle is zero. The direction in which the force acts is an important fact necessary for the specification. Therefore, the force is a vector quantity, and the resultant force of the forces must be obtained by the vector method. If we name the forces acting on the particles A, B, and C, the equilibrium state of the particles can be written in the form of the following equation.

$$R = A + B + C + \_ = 0$$

Where, R is the resultant of all the forces which applies on different entities.

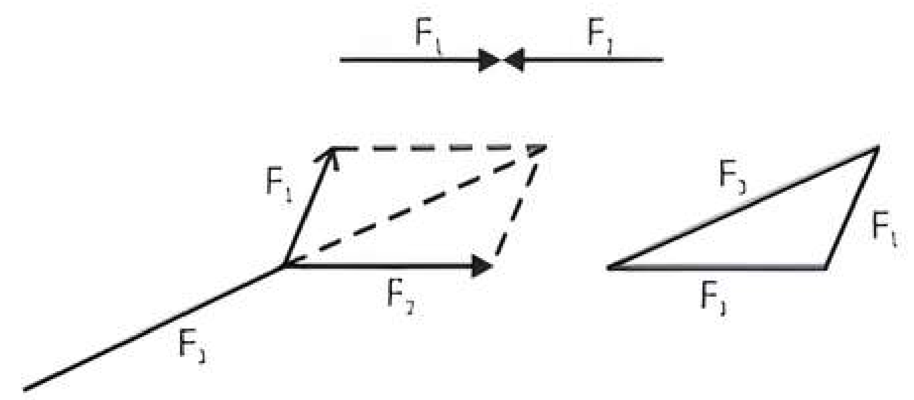
For Example, If two forces $F_1 = -F_2$,

acts on a different particle, then equilibrium requires,

$$F_1 = -F_2$$

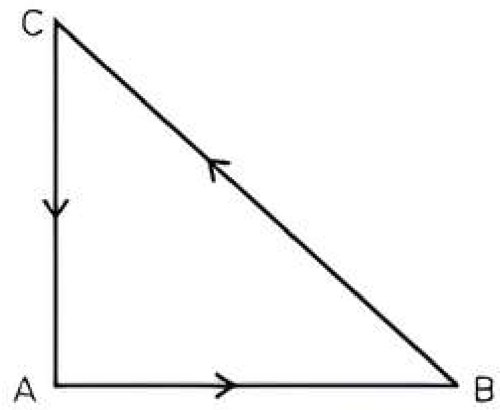
$$F_1 + F_2 = 0$$

$$F_1 + F_2 + F_3 = 0$$



**Example 1.13:** We have three forces which acts simultaneously on a particle which moves with velocity . All these forces are shown in magnitude and direction by all sides of a triangle ABC. The particle will now move with velocity will remain unchanged. What is the Resultant force of these forces?

**Ans.** Resultant force is zero, as three forces acting on the particle can be represented in magnitude and direction by three sides of a triangle in same order.




Hence, according to the Newton's 2nd law ($F = m\frac{dv}{dt}$), the velocity ($v$) of particle will be same.

## TOPIC 6

# COMMON FORCES IN MECHANICS

The gravitational force is all pervasive. Every object on the earth experiences the force of gravity due to earth. It is a non-contact force. All the other forces, common in mechanics, are contact forces. A contact force on an object arises due to contact with some other object, solid or fluid. When bodies are in contact (e.g., a book resting on a table, a system of rigid bodies connected by rods), there are mutual contact forces (for each pair of bodies) satisfying the third law of motion.

The component of contact force, normal to the surfaces in contact is called normal reaction. The component, parallel to the surfaces in contact is called friction.

**Tension force:** When a body of mass $m$ is fastened with the string, then the weight of the body acts downwards while a force acts just opposite to the downwards force for balancing the downwards force. This force known as force.

TOPIC 7

# FRICTION

Friction is the opposing force that arises when an object is actually in motion (sliding or rolling) or even trying to move across the surface of another object. Therefore, frictional force is the force that develops at the contact surface of two bodies and prevent (against) their relative motion.

The frictional force does not depend on the contact surface. Indeed, with the increase of the contact surface, the adhesive force also increases (by the same ratio), and the adhesive force responsible for the friction remains the same.

When the contact surfaces are extremely smooth, the intermolecular distances of the contact surface decrease, increasing the cohesive force between them. As a result, the binder pressure as well as the friction force increases.

## Static Friction, Limiting Friction and Kinetic Friction

The opposing force that occurs when one object tries to move on the surface of another object, which is in rest is called **static friction**.

**The limiting friction** is the maximum opposing force that acts just before one object moves over the surface of another.

**Kinetic friction or dynamic friction** is an opposing force that occurs when one body actually moves over the surface or has reached the limiting friction value, of another body.

Frictional force opposes (impending or actual) relative motion between two surfaces in contact. It is the component of the contact force along the common tangent to the surface in contact. Static friction $f_s$ opposes impending relative motion; kinetic friction opposes actual relative motion. They are independent of the area of contact and satisfy the following approximate laws:

$$f_s \le (f_s)_{max} = \mu_s R$$

$$f_k = \mu_k R$$

$\mu_s$ (coefficient of static friction) and $\mu_k$ (coefficient of kinetic friction) are constants characteristic of the pair of surfaces in contact. It is found experimentally that $\mu_k$ is less than $\mu_s$.

**Caution**

➥ *Students should know that kinetic friction is always slightly less than the limiting friction.*

**Example 1.14:** If a ladder weighing 250 N is placed against a smooth vertical wall having coefficient of friction between it and floor is 0.3, then what is the maximum force of friction available at the point of contact between the ladder and the floor?

(a) 75 N (b) 50 N
(c) 35 N (d) 25 N

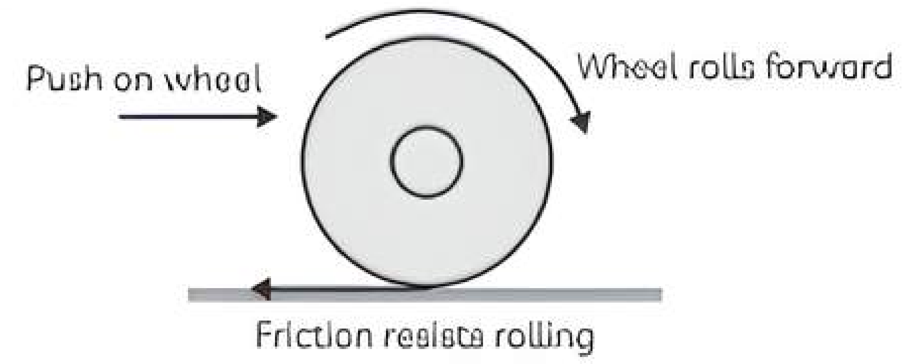
**Ans.** *(a) 75 N*

**Explanation:** Maximum force of friction:

$$F_s = \mu_s R$$
$$= 0.3 \times 250 = 75 \text{ N}$$

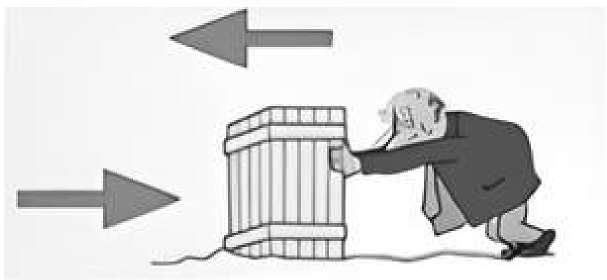
### Rolling Friction

The opposing force that occurs when an object rolls on the surface of another object is called rolling friction. Think of a wheel rolling on a road. As the wheel rolls on the road, it is slightly pressed against the road surface and slightly retracted, as shown in Figure.




*Rolling Friction*

**Sliding Friction:** Resists the motion of a pushed body so that only one surface of the body is in contact with the surface it is moving. This will happen when an object is pushed onto the table.



*Sliding Friction*

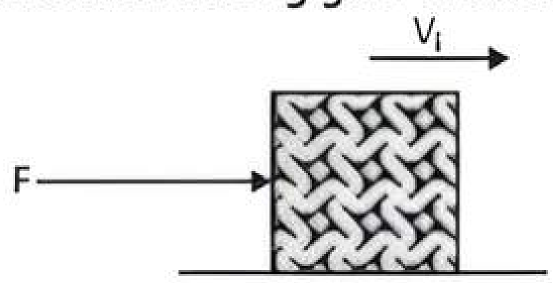
**Causes of rolling friction:** When an object rolls on a surface, the object deforms at the point of contact with the surface and the surface deforms at the point of contact with the object.

*Rolling Friction*

**Example 1.15:** Case Based:

Suppose at the local ice factory, a block of ice slides out of the freezer and a mechanical arm exerts a force to accelerate it across the icy, friction-free surface. Last week, the mechanical arm was malfunctioning and exerting pushes in a randomly directed factory. The various direction of forces applied to the moving block of ice are shown as:

(A) The force will speed up, when the force and the direction of motion of the ice block are in the same direction. Justify your answer.

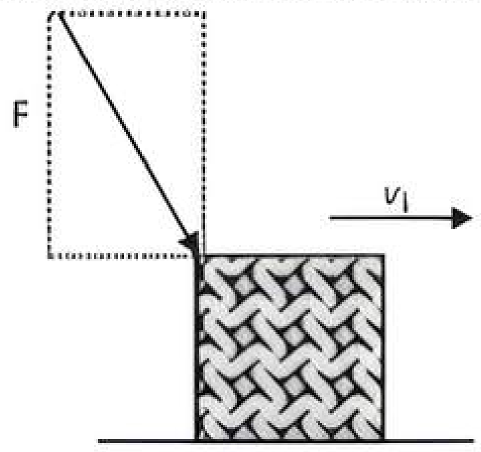



(B) What will happen when force and direction of motion of an ice block contact in opposite directions?

(C) When force applies diagonally on an ice block and motion of an ice block is in forward direction. Describe about the change in the speed of an ice block.

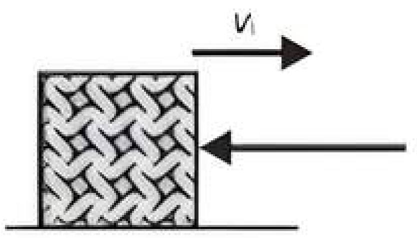
(D) Is there any effect on the speed of an ice block when force is applied on the top of an ice block and the motion of an ice block is in forward direction?

(E) What will happen when force is applied in comparison to motion as shown in figure?

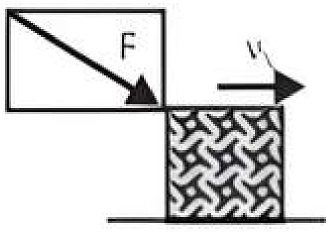



**Ans.** (A) There is an unbalanced force in the same direction as the block's motion is in the direction of motion of ice block. A force exerted in the direction of motion will cause an increase in speed.

(B) There is an unbalanced force in the opposite direction as the block's motion. A force-directed opposite an object's motion will cause a decrease in speed.




(C)




This unbalanced force has two components-one is downward and the other is rightward. Downward components of force cannot alter rightward speeds. But a rightward component of force would increase the rightward speed. A component of force exerted in the direction of motion will cause an increase in speed.

(D) There is no component of force in the direction of the motion. Thus, the object will neither speed up nor slow down. This downward component of force will only be counteracted by a greater normal force of the ground pushing up on the block. To change the speed of a moving object, there must be a component of force in the same direction or the opposite direction as the motion.

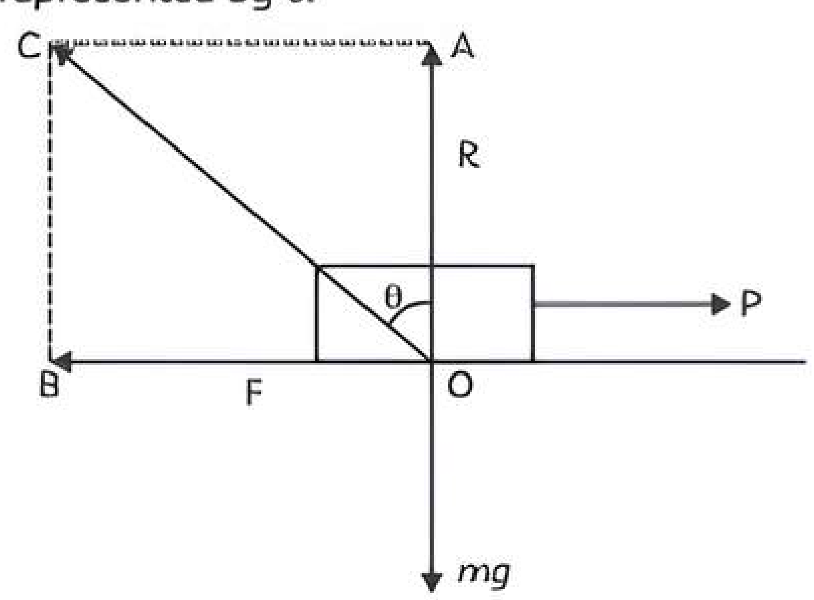
(E) This unbalanced force has two components: One is downward and the other is rightward. Downward components of force cannot alter rightward speeds. But a rightward component of force would increase the rightward speed. A component of force exerted in the direction of motion will cause an increase in speed.

## TOPIC 8

# ANGLE OF FRICTION

The angle of friction between any two surfaces in contact is defined as the angle, which is the resultant of the force of limiting friction F and normal reaction R, makes with the direction of normal reaction R.

It is represented by $\theta$.




In given fig., OA represents the normal reaction R which balances the weight $mg$ of the body. OB represents F, the limiting force of sliding friction when the body tends to move to the right. After complete the parallelogram OACB. OC represents the resultant of R and F.

By definition,

$\angle AOC = \theta$ is the angle of friction between the two bodies in contact.

The value of angle of friction depends on the nature of materials of the surfaces in contact and the nature of the surfaces.

**Relation between $\mu$ and $\theta$:**

In $\Delta AOC$, $\tan\theta = \dfrac{AC}{OA} = \dfrac{OB}{OA} = \dfrac{F}{R} = \mu$

Hence, $\mu = \tan\theta$

*i.e.*, coefficient of limiting friction between any two surfaces in contact is equal to the tangent of the angle of friction between them.

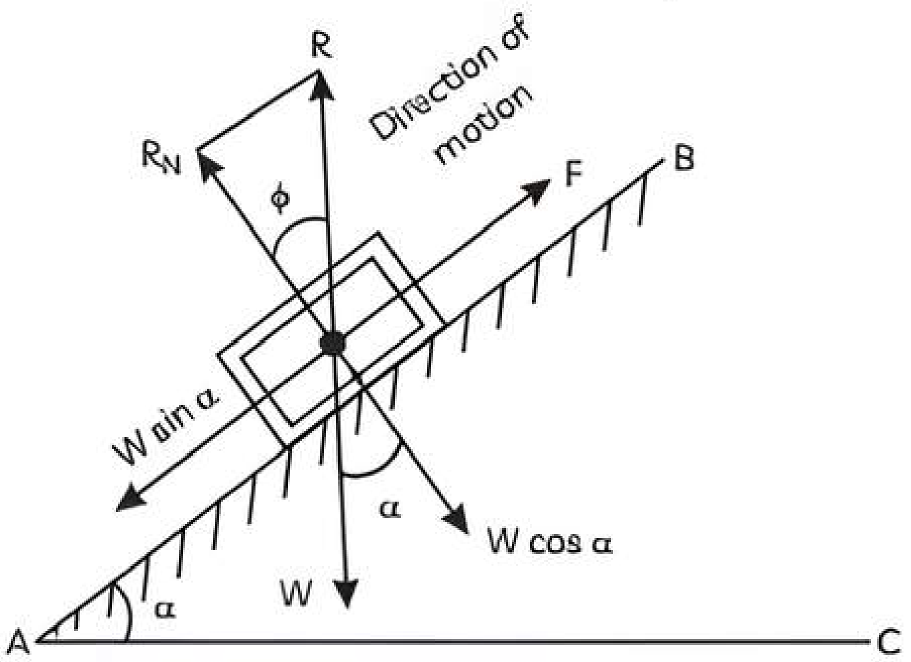
## Angle of Repose or Angle of Sliding

Angle of repose or angle of sliding is defined as the minimum angle of inclination of a plane with the horizontal, such that a body placed on the plane just begins to slide down.

It is represented by $\alpha$. Its value depends on material and nature of the surfaces in contact.

In given figure, AB is an inclined plane such that a body placed on it just begins to slide down.

$\angle BAC$, $\alpha$ = angle of repose




The various forces involved are:

Weight, $mg$ of the body, acting vertically downwards,

normal reaction R, acting perpendicular to AB,

Force of friction F, acting upto the plane AB.

Now, $mg$ can be resolved into two rectangular components: $mg \cos \alpha$ opposite to R and $mg \sin \alpha$ opposite to F.

In equilibrium,

$$F = mg \sin \alpha \quad \text{...(i)}$$

$$R = mg \cos \alpha \quad \text{...(ii)}$$

Dividing (i) by (ii), we get,

$$\mu = \tan \alpha$$

Hence, the coefficient of limiting friction between any two surfaces in contact is equal to the tangent of the angle of repose between them.

### Important

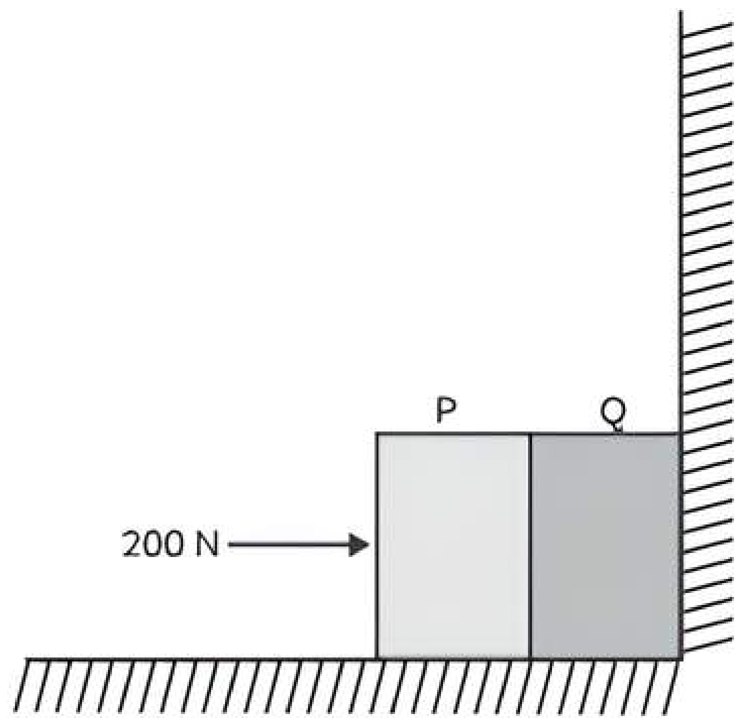
➥ *Combining the equation of angle of friction and angle of repose, we get.*

$$\theta = \alpha$$

$$\mu = \tan \theta = \tan \alpha$$

*Therefore, angle of friction is equal to angle of repose.*

**Example 1.16:** **Two bodies A and B of masses 5 kg and 10 kg in contact with each other rest on a table against a rigid wall (Fig.).**




**The coefficient of friction between the bodies and the table is 0.15. A force of 200 N is applied horizontally to P. What are (i) the reaction of the partition (ii) the action-reaction forces between P and Q? What happens when the wall is removed? Does the answer to (ii) change, when the bodies are in motion? Ignore the difference between $\mu_s$ and $\mu_k$.**

**Ans. (A)** When the wall exists and blocks P and Q are pushing the wall, there can't be any motion *i.e.*, blocks are at rest. Here,

(i) reaction of the partition = –(force applied on P) = 200 N toward left.

(ii) action-reaction forces between P and Q are 200 N each. A presses Q towards right with an action force 200 N and Q exerts a reaction force on P towards left having magnitude 200 N.

**(B)** When the wall is remove, motion can take place such that net pushing force provides the acceleration to block system. Hence, taking kinetic friction into account, we have

$$200 - \mu(m_1 + m_2)g = (m_1 + m_2)a$$

$$\Rightarrow \quad a = \frac{200 - \mu(m_1 + m_2)g}{(m_1 + m_2)}$$

$$= \frac{200 - 0.15 \times (5 + 10) \times 10}{(5 + 10)}$$

$$= \frac{200 - 22.5}{15}$$

$$= \frac{177.5}{15} = 11.8 \text{ ms}^{-2}$$

∴ If force exerted by P on Q be $F_{QP}$, then considering equilibrium (or free body diagram) of only block P, we have

$$200 - f_{k_1} = m_1 a + F_{QP}$$

or $$200 - \mu m_1 g = m_1 a + F_{QP}$$

$$F_{QP} = 200 - \mu m_1 g - m_1 a$$

$$= 200 - (0.15 \times 5 \times 10) - (5 - 11.8)$$

$$= 200 - 7.5 - 59$$

$$= 200 - 66.5 = 133.5 \text{ N} = 1.3 \times 10^2 \text{ N}$$

towards right

∴ Force exerted on P by Q,

$$F_{QP} = -F_{QP} = 1.3 \times 10^2 \text{ N towards left.}$$

**Lubrication**

The lubricants are the substances which are used to reduce the friction which acts between the two surfaces in contact, between which relative motion can occur. As the friction acts along the interface between two surfaces, the lubricants are applied there only. Hence they help to keep the parts smoothly moving. By reducing the friction, they basically reduce the loss of energy in the form of heat. As the energy losses are prevented, the overall efficiency of a machine is increased.

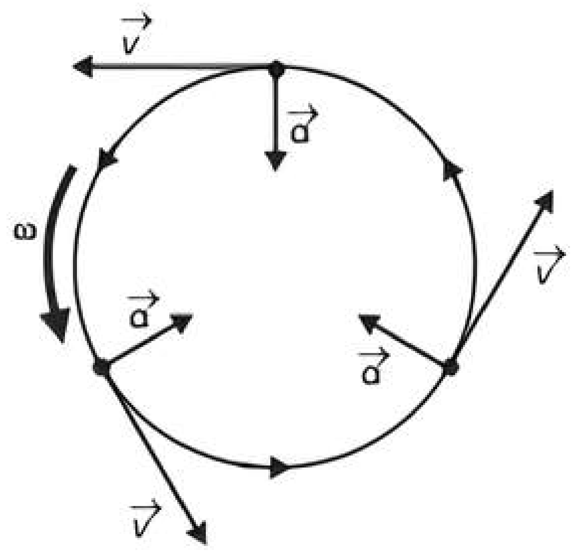
## TOPIC 9

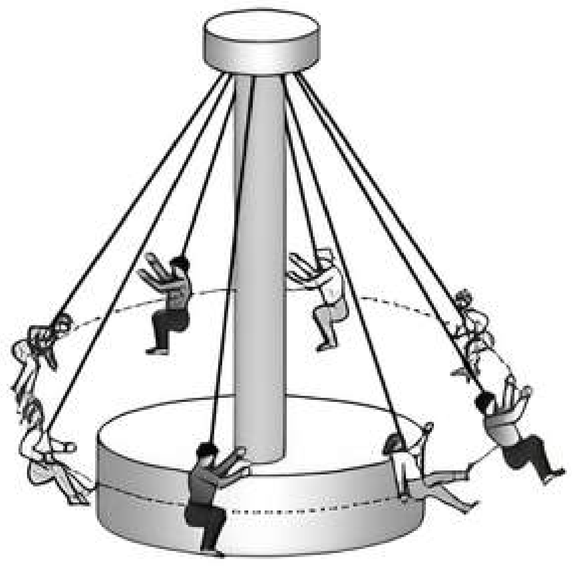
# DYNAMICS OF CIRCULAR MOTION OR CONCEPT OF CENTRIPETAL FORCE

## Centripetal Force

Centripetal force is the force required to move a body uniformly in a circle. This force acts along the radius and towards the center of the circle.

In fact, when a body moves in a circle, its direction of motion at any instant is along the tangent to the circle at that instant. From fig., we find that the direction of motion of the body moving in a circle goes on changing continuously.



According to Newton's first law of motion, a body cannot change its direction of motion by itself. An external force is required for this purpose. It is the external force, which is called the centripetal force.

On account of a continuous change in the direction of motion of the body, there is a change in velocity of the body, hence it undergoes an acceleration, called centripetal acceleration or radial acceleration.

Expression for centripetal force is:

*i.e.,* $$F = \frac{mv^2}{r} = mr\omega^2$$

**Example 1.17: A rotation is a transformation in a plane that turns every point of a preimage through a specified angle and direction about a fixed point. The fixed point is called the center of rotation. Can a centripetal force produce rotation?**

**Ans.** Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the $F_c$, the smaller the radius of curvature *r* and the sharper the curve. It can move a body along a circular path, but cannot produce rotational motion.

**Example 1.18: The phenomenon of raising the outer edge of the curved road above the inner edge is to provide necessary centripetal force to the vehicles to take a safer turn and the curved road is called Banking of Roads. Why this is required for the vehicles?**

**Ans.** When the circular road is banked, the horizontal component of the normal reaction of the road provides the necessary centripetal force for the vehicle to move it along the curved path.

## Centrifugal Force

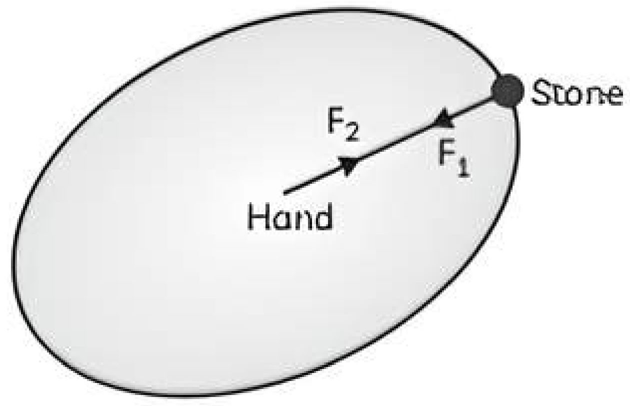
The natural tendency of a body is to move uniformly along a straight line. When we apply centripetal force on the body, it is forced to move along a circle. While moving actually along a circle, the body has a constant tendency to regain its natural straight line path. This tendency gives rise to a force called centrifugal force. Hence, Centrifugal force is a force that arises when a body is moving actually along a circular path, by virtue of tendency of the body to regain its natural straight line path.

Centrifugal forces can be referred as the reaction of centripetal force. As forces of action and reaction are always equal and opposite.

Therefore, the magnitude of centrifugal force = $\frac{mv^2}{r}$, which is the same as that of centripetal force.

However, the direction of centrifugal force is opposite to the direction of centripetal force, *i.e.*, centrifugal force acts along the radius and away from the center of the circle.

Note that, centripetal and centrifugal forces, being the forces of action and reaction, always act on different bodies. For rotated in a circle, centripetal force $F_1$ is applied on the stone by the hand. In turn, the hand is pulled outwards by centrifugal force $F_2$ acting on it, due to the tendency of the stone to regain its natural straight line path. The centrifugal and centripetal forces are shown in figure.




**Example 1.19: A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev./min in a horizontal plane. What is the tension in the string? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N?**

**Ans.** Mass of the stone, $m = 0.25$ kg

Radius of the circle, $r = 1.5$ m

Number of revolutions per second,

$$n = \frac{40}{60} = \frac{2}{3}\text{rps}$$

Angular Velocity, $\omega = \frac{v}{r} = 2\pi n$ —(i)

The centripetal force for the stone is provided by the tension T, in the string, *i.e.*,

$$T = F_{Centripetal} = \frac{mv^2}{r}$$

$$= mr\omega^2 = mr(2\pi n)^2$$

$$= 0.25 \times 1.5 \times \left(2 \times 3.14 \times \frac{2}{3}\right)^2$$

$$= 6.57 \text{ N}$$

Maximum tension in the string,

$$T_{max} = \frac{mv_{max}^2}{r}$$

Therefore,

$$v_{max} = \sqrt{\frac{T_{max} \times r}{m}}$$

$$= \sqrt{\frac{200 \times 1.5}{0.25}}$$

$$= \sqrt{1200} = 34.64 \text{ m/s}$$

Therefore, the maximum speed of the stone is 34.64 m/s.

## Motion of a Car on Level and Banked Road

### Motion of a Car on Level Road

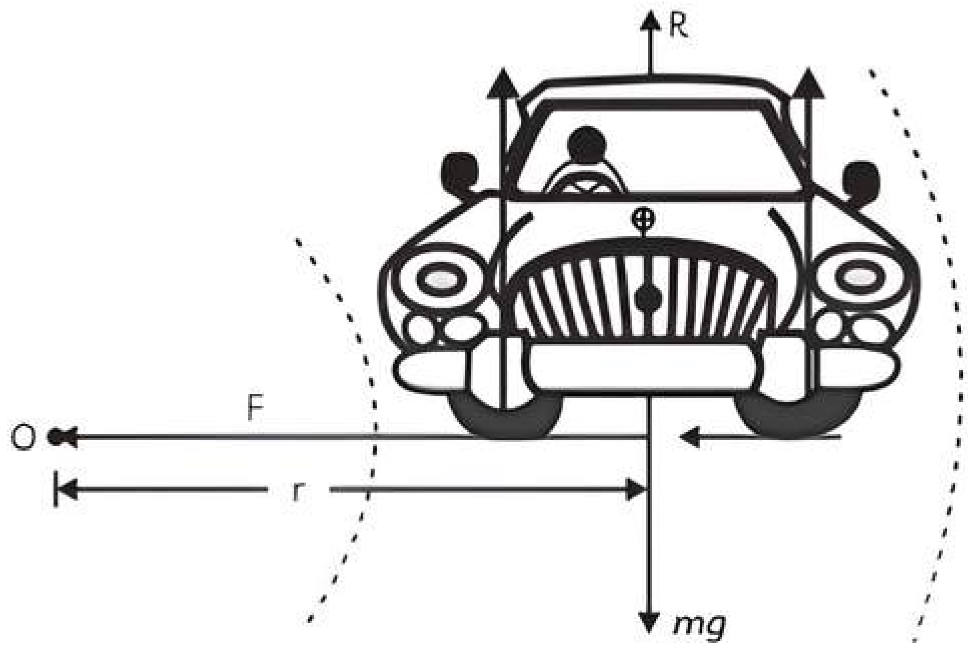
When a vehicle goes round a curved road, it requires some centripetal force. While rounding the curve, the wheels of the vehicle have a tendency to leave the curved path and regain the straight line path. The force of friction between the road and the tyres provided the centripetal force required to keep the car in motion around the curve.

There are three forces acting on the car such as:

(1) The weight of the car $mg$, acting vertically downwards.

(2) Normal reaction R of the road on the car, acting vertically upwards.

(3) Frictional force F, along the surface of the road, *i.e.*, towards the center of the turn.




As there is no vertical acceleration:

$$R = mg$$

The static friction opposes the impending motion of the car moving away from the circle. Thus,

$$\frac{mv^2}{r} \le F$$

$$F = \mu_s R = \mu_s mg$$

$\mu_s$ = coefficient of static friction between the road and the tyres.

$$\frac{mv^2}{r} \le \mu_s mg$$

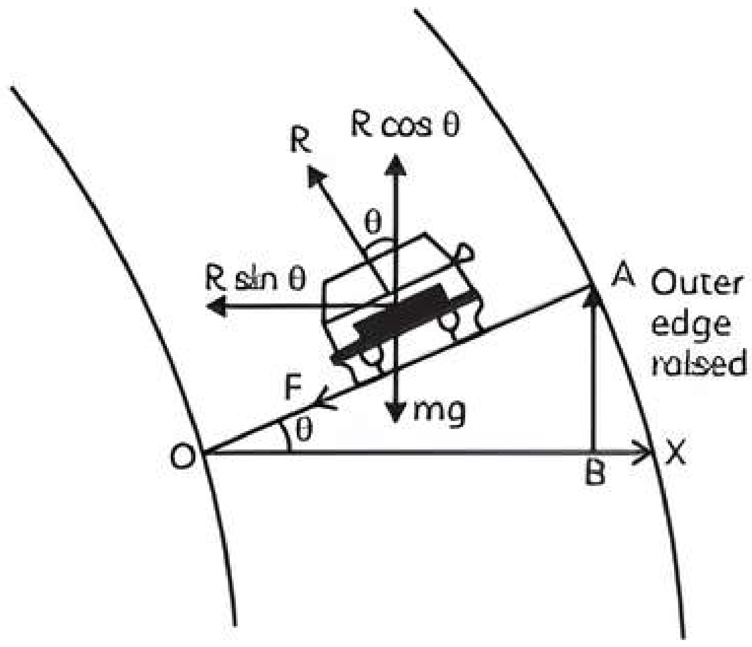
Or

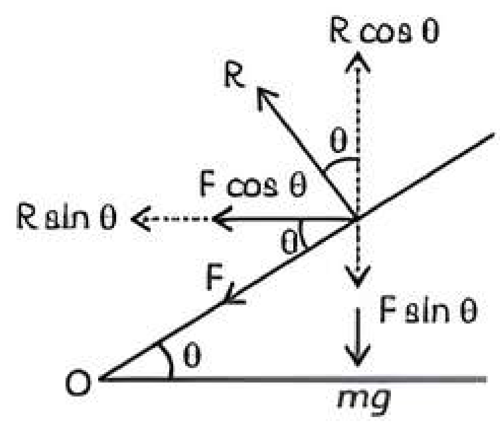
Maximum velocity of vehicle with which it can go around without skidding is,

$$v_{max} = \sqrt{\mu_s rg}$$

### Motion of a Car on Banked Road

Let us consider a car of weight $mg$ going along a curved path of radius $r$ with speed $v$ on a road banked at an angle θ, as shown in figure:




The forces acting on the car are:

(1) Weight, $mg$ acting vertically downwards.

(2) Normal reaction R acting upwards in a direction perpendicular to inclined plane making angle θ with the horizontal plane.

(3) Force of friction F acting downwards along the inclined plane because car tends to slip outwards. Reaction R can be resolved into two rectangular components.

   (i) θ, along vertically upward direction.

   (ii) θ, along the horizontal towards the center of the curved road. Force of friction F, can also be resolved into two rectangular components.

   - F cos θ, along the horizontal, towards the center of curved road.
   - F sin θ, along vertically downward direction.

Maximum velocity of vehicle with which, it can go around without skidding is,

$$v_{max} = \sqrt{\frac{rg(\mu_s + \tan\theta)}{(1 - \mu_s \tan\theta)}}$$

# OBJECTIVE Type Questions

[ 1 mark ]

## Multiple Choice Questions

**1. Swimming is possible on account of:**

(a) Third law of motion

(b) First law of motion

(c) Second law of motion

(d) Force of gravitation

**[Delhi Gov. SQP 2022]**

**Ans.** *(a) Third law of motion*

**Explanation:** Swimming is based on Newton's third law of motion because when we push the water, we feel equal force from the water on us and we move forward.

**2. A ball is moving with a constant velocity. After some time, it collides with a wall. Which one of the following remains conserved except momentum?**

(a) Energy (b) Displacement

(c) Power (d) Force

**Ans.** *(a) Energy*

**Explanation:** Apart from the momentum, the energy also remains conserved during a collision. As a matter of fact, the energy of a system often remains conserved. Displacement is defined as the change in the position of an object. Power is the amount of energy transferred or converted per unit time. A force is a push or pulls upon an object resulting from the object's interaction with another object

### Caution

- *Students must know that if a particle moves uniformly, means velocity is constant.*
  *(F = ma = m × 0 = 0)*
- *In the absence of external force, particle moves uniformly which is Newton's first law of motion that means we can derive mathematically Newton's first law of motion with the help of Newton's second law of motion.*

**3. The inherent property, with which a body resists any change in its state of motion, is known as:**

(a) Force (b) Momentum

(c) Inertia (d) Acceleration

**[Diksha]**

**Ans.** *(c) Inertia*

**Explanation:** A force is a push or pulls upon an object resulting from the object's interaction with another object. Momentum is defined as a product of the mass of a particle and its velocity. Momentum is a vector quantity; *i.e.*, it has both magnitude and direction. Inertia is the inherent property, with which a body resists any change in its state of motion. Acceleration is the rate of change of the velocity of an object with respect to time.

4. Inside the nucleus, two protons are held together by a force that overcomes the repulsion. This force is called:
(a) gravitational force
(b) electrostatic force
(c) weak force
(d) strong force.

**Ans.** *(d) strong force.*

**Explanation:** The force of attraction between all masses in the universe. Electrostatic forces are non-contact forces; they pull or push on objects without touching them. Weak interaction, also called weak force or weak nuclear force, a fundamental force of nature that underlies some forms of radioactivity, governs the decay of unstable subatomic particles such as mesons and initiates the nuclear fusion reaction that fuels the Sun. The strong force, a fundamental interaction of nature that acts between subatomic particles of matter. The strong force, a fundamental interaction of nature that acts between subatomic particles of matter.

5. A force is one that acts on an object for only a short time and is primarily produced in a collision that results in a change in the velocity or momentum of one or more of the objects involved in the collision. Which among the following forces exert this property?
(a) Tension force
(b) frictional force
(c) Impulsive forces
(d) Contact force

**Ans.** *(c) Impulsive forces*

**Explanation:** The force that is transmitted through a rope, string, or wire when pulled by forces acting from opposite sides. The frictional force is the opposing force that is created between two surfaces that try to move in the same direction or that try to move in opposite directions. The force which acts on an object for a short period of time is known as Impulsive force. Contact force can be seen as frictional force that acts while driving a car or while river rafting or ice-skating.

6. In order to raise a mass of 100 kg, a man of 60 kg fastens the rope to it, and passes the rope over a smooth pulley. He climbs the rope with an acceleration $\frac{5g}{4}$ relative to rope. The tension in the rope is:
(Take $g = 10$ m/s$^2$)
(a) 1200 N (b) 1218 N
(c) 928 N (d) 1152 N [Diksha]

**Ans.** *(b) 1218 N*

**Explanation:** Let T be the tension in the rope and $a$ is the acceleration of that rope. The absolute acceleration of the man will be;

$$\left(\frac{5g}{4} - a\right)$$

Equation of motion for mass gives:

$$T - 100g = 100a$$

and equation of motion for man gives:

$$T - 60g = 60$$

by solving above equations, we get,

$$T = 1218 \text{ N}$$

7. A ball is travelling with uniform translator motion. This means that,
(a) It is at rest.
(b) The path can be a straight line or circular and the ball travels with uniform
(c) All parts of the ball have the same velocity (magnitude and direction) and the velocity is constant.
(d) The center of the ball moves with constant velocity, and the ball spins about its center uniformly. [NCERT Exemplar]

**Ans.** *(c) All parts of the ball have the same velocity (magnitude and direction) and the velocity is constant.*

**Explanation:** When a body moves in such a way that the linear distance covered by each particle of the body is the same during the motion, then the motion is said to be translatory or translation motion.

**Caution**

- *Students should know that the momentum of the system is conserved and not that of the individual particles. The momentum of the individual bodies in the system might increase or decrease according to the situation, but the momentum of the system will always be conserved, as long as there is no external net force acting on it.*

8. During inelastic collision between two bodies, which of the following quantities always remain conserved?
(a) Total kinetic energy
(b) Total potential energy
(c) Total linear momentum
(d) None of the above [Delhi Gov. SQP 2022]

**Ans.** *(c) Total linear momentum*

**Explanation:** Because no external forces occur on the colliding bodies during the collision, total linear momentum is always preserved in all collisions, but kinetic energy is not conserved in all collisions.

Kinetic energy is preserved only in fully elastic collisions, while it is lost in inelastic collisions. In an inelastic collision, total kinetic energy is not preserved.

**9. One end of a string of length $l$ is connected to a ball mass $m$ and the other to a small peg on a smooth horizontal table. If the ball moves in a circle with speed $v$, the net force on the ball (directed towards the center) is:**

(a) T (b) $T-\frac{mv^2}{l}$

(c) $T+\frac{mv^2}{l}$ (d) 0 [Diksha]

**Ans.** *(a) T*

**Explanation:** when a ball connected a string revolves in a circle, the centripetal force is provided by tension produced in that string. So, the total force on the ball will be equal to tension, *i.e.*,

$$F=T=\frac{mv^2}{l}$$

**10. A body whose momentum is constant must have constant:**

(a) velocity

(b) force

(c) acceleration

(d) none of the above [Delhi Gov. SQP 2022]

**Ans.** *(a) velocity*

**Explanation:** For a given mass $P \propto v$.

If momentum is constant, then velocity must also be constant.

**11. A body of mass $m$ is placed on a rough surface with coefficient of friction which is inclined at an angle of θ. If the mass is in equilibrium, then**

(a) $\theta = \tan^{-1}\mu$ (b) $\theta=\tan^{-1}\left(\frac{1}{\mu}\right)$

(c) $\theta=\tan^{-1}\left(\frac{m}{\mu}\right)$ (d) $\theta=\tan^{-1}\left(\frac{\mu}{m}\right)$

[Diksha]

**Ans.** *(a) $\theta = \tan^{-1}\mu$*

**Explanation:** Given body of mass = $m$

At the equilibrium point, $F = mg \sin\theta$ (in case of limiting condition) ...(i)

Normal force will be, $R = mg\cos\theta$ ...(ii)

Now compare both equation (i) and (ii).

$$\frac{F}{R}=\frac{mg\sin\theta}{mg\cos\theta}$$

Frictional force,

$$F = \mu R$$

$$\therefore \quad \mu = \tan\theta$$

$$\theta = \tan^{-1}(\mu)$$

This is the maximum value of θ for mass $m$, to be in rest.

**12. Three blocks of masses $m$, $3m$ and $5m$ are connected by strings. After an upward force F is applied on block $m$, the masses move in the upward direction at constant speed $v$. What is the net force on the block of mass $3m$?**

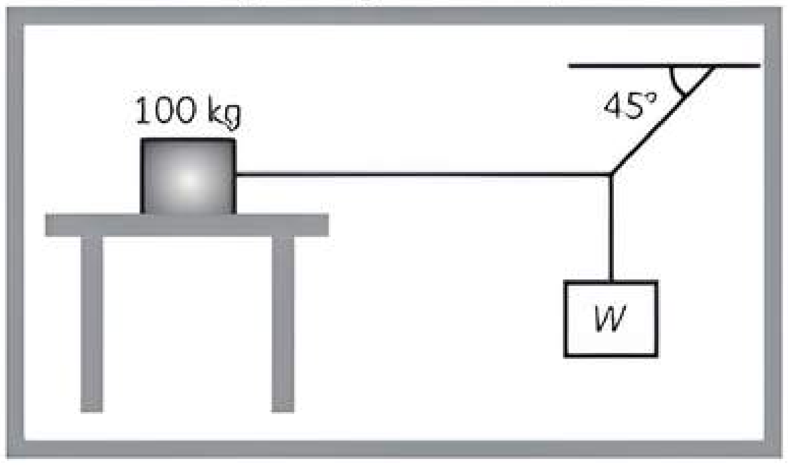
(a) zero (b) $3\,mg$

(c) $5\,mg$ (d) $8\,mg$ [Diksha]

**Ans.** *(a) zero*

**Explanation:** As all blocks are moving with constant velocity, means zero acceleration. Therefore, net force on all blocks will be zero.

**13. The system shown in the figure is in equilibrium. The maximum value of W, so that the maximum value of static frictional force on 100 kg. body is 450 N, will be:**




(a) 100 N (b) 250 N

(c) 450 N (d) 1000 N

**Ans.** *(c) 450 N*

**Explanation:** For vertical equilibrium,

$$T_1 \sin 45° = W$$

Therefore, $$T_1=\frac{W}{\sin 45°}$$

For horizontal equilibrium,

$$T_2 = T_1\cos 45^0$$

$$=\frac{W}{\sin 45°}\cos 45° = W$$

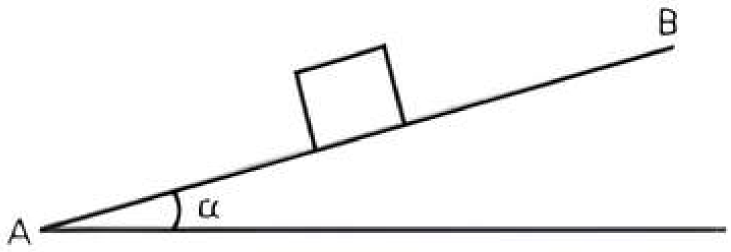
and for the critical condition,

$$T_2 = F$$

Therefore, $$W = T_2$$

$$F = 450\text{ N}$$

**14. Consider the figure given below in which a particle of weight W resting a smooth (frictional less) inclined plain AB with the help of force F acting on the particle at angle θ with the line AB.**



Find the force F and normal reaction:

(a) $\frac{(W\cos\theta)}{\sin\alpha}, \frac{[W\cos(\alpha-\theta)]}{\cos\theta}$

(b) $\frac{(W\sin\alpha)}{\cos\theta}, \frac{[W\sin(\alpha+\theta)]}{\cos\theta}$

(c) $\frac{(W\sin\alpha)}{\cos\theta}, \frac{[W\sin(\alpha+\theta)]}{\cos\theta}$

(d) none of the above

**Ans.** *(b)* $\frac{(W\sin\alpha)}{\cos\theta}, \frac{[W\sin(\alpha+\theta)]}{\cos\theta}$

**Explanation:** In figure given below shows all the forces acting on the block.

Using law of Sine,

$$\frac{F}{\sin(180-\alpha)} = \frac{N}{\sin(90+\alpha+\theta)} = \frac{W}{\sin(90-\theta)}$$

Solving this,

$$F = \frac{(W\sin\alpha)}{\cos\theta},\ N = \frac{[W\cos(\alpha+\theta)]}{\cos\theta}$$

**15.** **A particle of mass M, originally at rest is subjected to a force whose direction is constant but whose magnitude varies with the time according to the relation;**

$$F = F_0\left[1-\left(\frac{t-T}{R}\right)^2\right]$$

**Where, $F_0$ and T are constant. The force acts only for the time interval 2T. Find the velocity $v$ of the particle after time 2T.**

(a) $F_0/3M_0$ (b) $4F_0/3M$

(c) $F_0/2M_0$ (d) none of these

**Ans.** *(b)* $4F_0/3M$

**Explanation:**

$$F = F_0\left[1-\left(\frac{t-T}{R}\right)^2\right]$$

$$M\frac{dv}{dt} = F_0\left[1-\left(\frac{t-T}{R}\right)^2\right]$$

$$\frac{dv}{dt} = \frac{F_0}{M}\left[1-\left(\frac{t-T}{R}\right)^2\right]$$

$$\int dv = \frac{F_0}{M}\int_0^{2T}\left[1-\left(\frac{t-T}{R}\right)^2\right]dt$$

$$v = \frac{F_0}{M}\left[t-\frac{T}{3}\left(\frac{t}{T}-1\right)^3\right]_0^2$$

$$= \frac{4F_0}{3M}$$

**16.** **Find out the time through which the object will come to rest.**

(a) $t \to \infty$ (a) $t = kt/m$

(a) $t = mt/k$ (a) none of these

[Diksha]

**Ans.** *(a)* $t \to \infty$

**Explanation:** $F = -kv$

$$m\frac{dv}{dt} = -kv$$

$$\frac{dv}{v} = -kdt/m$$

On Integrating:

$$v = \frac{-kt}{m} + c$$

now $t = 0$, $v = v_0$

So, $v = v_0 \exp. \frac{-kt}{m}$

So, at $t \to \infty$

$$v = 0$$

**17.** **A bullet of mass $m$ moving with a speed $v$ strikes a wooden block of mass M & gets embedded into the block. The final speed is:**

(a) $\frac{m}{m+M}v$ (b) $\frac{m}{2m+M}v$

(c) $\frac{m}{m+2M}v$ (d) $\frac{2m}{m+M}v$

[Delhi Gov. SQP 2022]

**Ans.** *(a)* $\frac{m}{m+M}v$

**Explanation:**

*Let, $m$ = mass of the bullet*

M = mass of the wooden block which is at rest initially

$v$ = velocity of the bullet

$v'$ = velocity of the bullet-block system

From the law of conservation of momentum.

$$mv = (m + M)v'$$

$$v' = \frac{m}{m+M}v$$

**18.** **A car of mass $m$ starts from rest and acquires a velocity along east ($v > 0$) in two seconds. Assuming the car moves with uniform acceleration, the force exerted on the car is**

(a) eastward and is exerted by the car engine.

(b) Eastward and is due to friction on the tires exerted by the road.

(c) More than eastward exerted due to the engine and overcomes the friction of the road.

(d) exerted by the engine.

**Ans.** (c) *More than eastward exerted due to the engine and overcomes the friction of the road.*

**Explanation:** Let us assume the eastward direction as $x$-axis. A car is able to move towards due to friction acting between its tyres and the road. The force of friction of the road on the tyre acts in the forward direction and is equal but in the opposite direction to the force of friction of the tyre on the road.

Mass of the car = $m$

As car starts from rest, its initial velocity.

$u = 0$ Velocity acquired along east = $v\hat{i}$

Time interval (in which car acquired that velocity), $t = 2$ s.

As acceleration is uniform, so by applying kinematic equation ($v = u + at$),

we get $v = u + at$

$$v\hat{i} = 0 + a \times 2$$

$$a = \frac{v}{2}\hat{i}$$

$$F = ma = \frac{mv}{2}\hat{i}$$

Hence, the force acting on the car is $\frac{mv}{2}$ towards east. As external force on the system is only friction, hence, the force $\frac{mv}{2}$ is applied by friction. Hence, force by engine is internal force.

**Caution**

- *Students must know that if a body moves in a negative direction, the value will not be taken in negative as a physical quantity cannot have a negative value. Therefore, a modulus should be taken of final answer to convert it into a positive value.*

**19.** **If all the matters were made of electrically neutral particle such as neutrons:**

**(I) there would be no force of friction.**

**(II) there would be no tension in the string.**

**(III) it would not be possible to sit on a chair.**

**(IV) the earth could not move around the sun.**

**Choose the correct option:**

**(a) (I), (II), (III)** **(b) (II), (III), (IV)**

**(c) (I), (II), (IV)** **(d) (I), (III), (IV)**

**Ans.** (a) *(I), (II), (III)*

**Explanation:** When there are no charged particles and only neutral particles, then there would be no interaction between the particles or molecules or atoms and we would not get any solid structure or lattice. So, the main cause of friction would be absent. So, there would be no force of friction. There would be no tension in the string as the string would collapse because of the absence of a solid structure and it would not be possible to sit on a chair as well.

Friction, down to atomic level is because of electromagnetic forces between positive and negative charges and the earth moves around the sun because of gravitational force.

**20.** **If a cork of mass 10 g floating on the water, then the net force will be:**

**(a) 10 N** **(b) more than 10 N**

**(c) less than 10 N** **(d) zero**

**Ans.** (d) *zero*

**Explanation:** The weight of the cork is acting downward direction, but it is balanced by the buoyant force which is exerted by the water in the upward direction. Hence, no net force is acting on floating cork.

## Assertion-Reason Questions

**Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these question from the codes (a), (b), (c) and (d) as given below:**

**(a) Both A and R are true and R is the correct explanation of A.**

**(b) Both A and R are true and R is not the correct explanation of A.**

**(c) A is true but R is false.**

**(d) A is false and R is also false**

**21.** **Assertion (A): A table cloth can pulled from a table without disloding the dishes.**

**Reason (R): To every action there is an equal and opposite reaction.**

**[Delhi Gov. QB 2022]**

**Ans.** (b) *Both A and R are true and R is not the correct explanation of A.*

**Explanation:** Newton's first law states that when a cloth is dragged off a table, it enters a state of motion, while the dishware stays motionless because of the law of initial. Therefore, the disher doesn't move when we remove the cloth from the table.

**22.** **Assertion (A): A rocket works on the principle of conservation of linear momentum.**

**Reason (R): For two bodies system when there is a change in momentum of one body, the same change occurs in the momentum of the second body but in the opposite direction.**

**[Diksha]**

**Ans.** (a) *Both A and R are true and R is the correct explanation of A.*

**Explanation:** As the fuel in the rocket undergoes combustion, the gases so produced leave the body of the rocket with large velocity and give upthrust to the rocket. If we assume that the fuel is burnt at a constant rate, then the rate of change of momentum of the rocket will be constant. As more and more fuel is burnt, the mass of the rocket goes on decreasing and it leads to increase of the velocity of rocket more and more rapidly.

**23. Assertion (A):** **A particle is thrown vertically upwards. If air resistance is taken into consideration, then retardation in upward journey is more than the acceleration in downward journey.**

**Reason (R):** **Some mechanical energy is lost in the form of heat due to air friction**

**Ans.** *(b) Both A and R are true and R is not the correct explanation of A.*

**Explanation:** Retardation in upward journey,

$$a_1 = \frac{w+F}{m}$$

Acceleration in downward journey,

$$a_2 = \frac{w-F}{m}$$

Therefore, $a_1 > a_2$.

**24. Assertion (A):** **It is difficult to move a cycle along the road with brakes on.**

**Reason (R):** **Sliding friction is greater than rolling friction.**

**[Delhi Gov. QB 2022]**

**Ans.** *(a) Both A and R are true and R is the correct explanation of A.*

**Explanation:** When the brakes are applied, the wheels skid rather than rolling. Rolling friction is less than sliding friction. Therefore, moving a bicycle down the road when its brakes are engaged is challenging.

**25. Assertion (A):** **Friction always opposes motion of a body.**

**Reason (R):** **Without friction also, one can move on a smooth surface.**

**Ans.** *(d) A is false and R is also false.*

**Explanation:** Friction opposes the relative motion of the bodies in contact. By throwing something backwards, you can move forward.

**26. Assertion (A):** **It is difficult to move a cycle along the road with its brakes on.**

**Reason (R):** **Sliding friction is greater than rolling friction.**

**Ans.** *(a) Both A and R are true and R is the correct explanation of A.*

**Explanation:** When brakes are on there is no rolling of the wheels and the wheels slide. The sliding friction is greater than the rolling friction. Thus, it is difficult to move a cycle along the road with its brakes on.

**27. Assertion (A):** **Centripetal force is always required for motion in curved path.**

**Reason (R):** **On a banked curved track, vertical component of normal reaction provides the necessary centripetal force.**

**[Delhi Gov. QB 2022]**

**Ans.** *(d) A is false and R is also false.*

**Explanation:** The horizontal portion of a normal response supplies centripetal force. The vehicle's vertical component balances its weight. For turning, a centripetal force is necessary.

**28. Assertion (A):** **Friction opposes motion of a body.**

**Reason (R):** **Static friction is self-adjusting, while kinetic friction is constant.** **[Diksha]**

**Ans.** *(d) A is false and R is also false.*

**Explanation:** Friction opposes the relative motion of the bodies in contact, not the motion.

**29. Assertion (A):** **Mass is a measure of inertia of the body in linear motion.**

**Reason (R):** **Greater the mass, greater is the force required to change its state of rest or of uniform motion.**

**Ans.** *(a) Both A and R are true and R is the correct explanation of A.*

**Explanation:** Inertia is the resistance of any physical object to any change in its state of motion. It is the parameter used to describe the motion of the object and how it is affected by the applied force. And the mass ($m$) is the measure of it.

Again $$\vec{F} = m\frac{d\vec{v}}{dt}.$$

So, force is proportional to mass. So, greater the mass, the greater is the force required to change its state of rest or of uniform motion.

# CASE BASED Questions (CBQs)

[ 4 & 5 marks ]

**Read the following passages and answer the questions that follow:**

**30.** *The product of force and time which is the change in momentum of the body remains a measurable quantity. This product is called impulse.*

*Impulse = Force × time duration*

*= Change in momentum*

*Large force acting for a short time to produce a finite change in momentum is called an impulsive force.*

*The third law of motion states that when one object exerts a force on another object, the second object instantaneously exerts a force back on the first. These two forces are always equal in magnitude but opposite in direction. These forces act on different objects and never on the same object. The two opposing forces are also known as action and reaction forces.*

**(A) A bird is sitting on the floor of a wire cage and the cage is in the hand of a boy. The bird starts flying in the cage. Will the boy experience any change in the weight of the cage?**

**(B) A nucleus is at rest in the laboratory frame of reference. Show that if it disintegrates into two smaller nuclei, the products must move in opposite directions.**

**(C) A ball of mass $m$ strikes a wall at an angle of 30° with velocity $v$ and is reflected at the same angle without any loss of speed. What is the magnitude of impulse imparted to the ball by the wall and what is the direction of the force of the wall due to the ball?**

**Ans.** (A) When the bird starts flying inside the cage the weight of the bird is no more experienced as air inside is in free contact with atmospheric air. Hence, the cage will appear lighter.

(B) Let $m_1$ and $m_2$ be the respective masses of the parent nucleus and the two daughter nuclei. The parent nucleus is at rest.

Initial momentum of the system (parent nucleus) = 0

Let $v_1$ and $v_2$ be the respective velocities of the daughter nuclei having masses $m_1$ and $m_2$.

Total linear momentum of the system after disintegration = $m_1 v_1 + m_2 v_2$

According to the law of conservation of momentum:

Total initial momentum = Total final momentum

$$0 = m_1 v_1 + m_2 v_2$$

$$v_1 = \frac{-m_2 v_2}{m_1}$$

Here, the negative sign indicates that the fragments of the parent nucleus move in directions opposite to each other.

(C) $$\vec{p}_i = mv \sin 30° \hat{i} - mv \cos 30° \hat{j}$$

$$\vec{p}_i = mv \sin 30 \hat{i} - mv \cos 30 \hat{j}$$

$$\Delta \vec{p} = \vec{p}_f - \vec{p}_i$$

$$= -mv \sin 30° \hat{i} = -mv \hat{i}$$

$$|\Delta \vec{p}| = -mv$$

Negative sign of the impulse shows that it is along a negative $x$-direction. Since impulse and force are in the same direction, the force on the ball is along the negative direction of the $x$-axis. Hence, the force on the wall will be along the positive $x$-axis.

**31.** *When two bodies are in contact, each experiences a contact force by the other. The component of the contact force parallel to the surfaces in contact, which opposes impending or actual relative motion between the two bodies in contact is opposed by static friction. Kinetic friction opposes actual relative motion between two bodies in contact. There is a yet another type of friction which opposes rolling motion of one body over the surface of another body. It is called rolling friction. We often regard friction as something undesirable. However in many practical situations friction is critically needed.*

**(A) What is the direction of friction?**

**(a) Friction always acts tangential to the surface in contact.**

**(b) Friction acts normal to the surface in contact.**

**(c) Direction depends upon weight of body which moves over surface of another body.**

**(d) None of these.**

(B) Which one of the following statement is not correct about friction?

(a) Friction is an self adjusting force.

(b) Force of friction is independent of area of contact as long as normal reaction remains same.

(c) Sliding friction is greater than static friction.

(d) Limiting friction is the maximum static friction.

(C) An automobile is moving on a horizontal road with a speed $v$. If the coefficient of friction between the tyres and the road is $\mu$. What is the shortest distance in which the automobile can be stopped?

(a) $\frac{v^2}{\mu g}$ (b) $\frac{2v^2}{\mu g}$

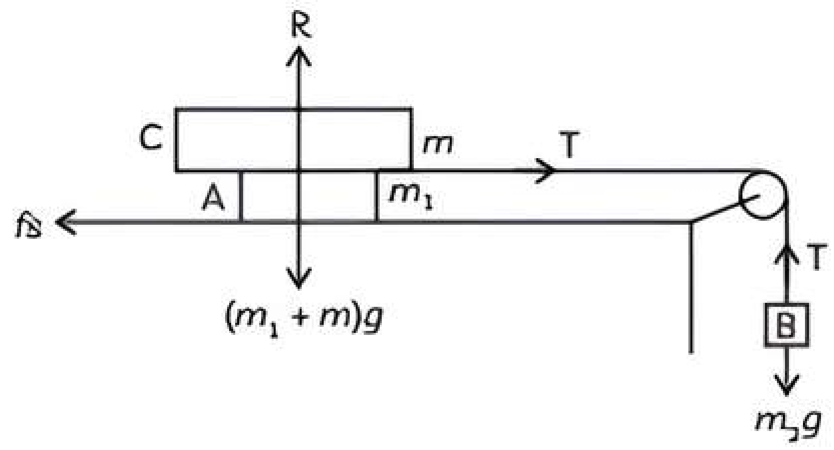
(c) $\frac{v^2}{4\mu g}$ (d) $\frac{v^2}{2\mu g}$

(D) What will be the maximum acceleration of the train in which a box lying on the floor will remain stationary? Given that the coefficient of friction between the box and trains floor is 0.15. ($g = 10$ m/s$^2$)

(a) 2 m/s$^2$ (b) 2.5 m/s$^2$

(c) 1 m/s$^2$ (d) 1.5 m/s$^2$

(E) In figure, the masses of blocks A & B are 10 kg and 15 kg. Calculate the minimum mass of C which may stop A from slipping. Coefficient of friction between block A and table is 0.2.




(a) 5 kg (b) 15 kg

(c) 25 kg (d) 35 kg

**Ans.** (A) (a) *friction always acts tangential to the surface in contact.*

**Explanation:** When two surfaces are in contact and are moving relative to each other, the frictional force acts in the tangential direction to the surface of contact.

(B) (c) *Sliding friction is greater than static friction.*

**Explanation:** While sliding, the points of contact between two surfaces do not get enough time to get interlocked, whereas more interlocking takes place when the surfaces are not moving over each other. Therefore, sliding friction is less than static friction.

(C) (d) $\frac{v^2}{2\mu g}$

**Explanation:** Let '$m$' be the mass of the vehicle. So, frictional force experienced by the vehicle on its tyres is $f = \mu mg$

So, the acceleration of the vehicle due to this frictional force is, $a = -\mu g$ (acceleration is directed opposite to the motion of the vehicle)

Now using,

$$v^2 = u^2 + 2as$$

$$\Rightarrow \quad 0 = v^2 - 2\mu gs$$

[initial velocity = $v$, final velocity = 0]

So, $$s = \frac{v^2}{(2\mu g)}$$

(D) (d) 1.5 m/s

**Explanation:** Given, $\mu = 0.15$, $g = 10$

$$a_{max} = \mu_s g$$
$$= 0.15 \times 10$$
$$= 1.5 \text{ ms}^{-2}$$

(E) (b) 15 kg

**Explanation:** Let the mass of block C is m

For block A,

$$R = (m_1 + m)$$
$$T = f_s = \mu R = \mu(m + m_1)g$$

To avoid slippling of A, for block B

$$T = m_2 g$$

or $$\mu(m+m_1) = m_2 g$$

$$0.2(m + 10) = 5$$

$$\Rightarrow \quad m = 15 \text{ kg}$$

**32.** *The first law of motion refers to the simple case when the net external force on a body is zero. The second law of motion refers to the general situation when there is a net external force acting on the body.*

*It relates the net external force to the acceleration of the body.*

*The following common experiences indicate the importance of momentum for considering the effect of force on motion.*

*Suppose a light-weight vehicle (say a small car) and a heavy weight vehicle (say a loaded truck) are parked on a horizontal road. We all know that a much greater force is needed to push the truck than the car to bring them to the same speed at the same time. Similarly, a greater opposing force is needed to stop a heavy body than a light body at the same time, if they are moving with the same speed.*

*Speed is another important parameter to consider. A bullet fired by a gun can easily pierce human tissue before it stops, resulting in casualty. The same bullet fired with moderate* *speed will not cause much damage. Thus, for a given mass, the greater the speed, the greater is the opposing force needed to stop the body in a certain time.*

*The greater the change in the momentum in a given time, the greater is the force that needs to be applied.* [Delhi Gov. SQP 2022]

(A) Momentum of a body is defined to be the product of:
- (a) its mass and velocity square
- (b) its mass and acceleration
- (c) its mass and velocity
- (d) its mass and applied force

(B) The rate of a change of a particle's momentum $p$ is given by the force acting on the particle, refers to:
- (a) Newton's first law of motion
- (b) Newton's second law of motion
- (c) Newton's third law of motion
- (d) None of the above

(C) A bullet of mass 0.04 kg moving with a speed of 90 $ms^{-1}$ enters a heavy wooden block and is stopped after a distance of 60 cm. The average resistive force exerted by the block on the bullet is:
- (a) 270 N (b) 450 N
- (c) 375 N (d) 540 N

(D) The motion of a particle of mass $m$ is described by $y = ut + \frac{1}{2}gt^2$. The force acting on the particle will be:
- (a) $\frac{mg}{4}$ (b) $\frac{mg}{2}$
- (c) $mg$ (d) $4\ mg$

(E) A rope of length 10 m and linear density 0.5 kg/m is lying length wise on a smooth horizontal floor. It is pulled by a force of 25 N. The tension in the rope at a point 6 m away from the point of application is:
- (a) 5 N (b) 10 N
- (c) 15 N (d) 20 N

**Ans.** (A) *(c) its mass and velocity*

**Explanation:** $p = m \times v$

Momentum is the product of mass and velocity. It is a vector quantity having both magnitude and direction. The unit of momentum in SI unit is $Kg.ms^{-1}$.

(B) *(b) Newton's second law of motion*

**Explanation:** Because the law of conservation of momentum may be deduced from the concept of action and reaction, which asserts that every force has a reciprocating equal and opposite force, it is based on Newton's third law. When you push against a wall, it pushes back with the same amount of force.

(C) *(a) 270 N*

**Explanation:** The mass of the bullet is $m = 0.04$ kg.

The initial speed of the bullet is $u = 90$ m/s

The final velocity of the bullet is $v = 0$

The bullet will stop at a distance is $s = 60$ cm

Now,

$$v^2 = u^2 + 2as$$

or $$a = \frac{-u^2}{2s} = \frac{-(90)^2}{2 \times 0.6} = -6750$$

or $$a = -6750 \text{ m/s}^2$$

Again,

$$F = ma$$

or $$F = 0.04 \times (-6750) = -270$$

or $$F = -270 \text{ N}$$

(D) *(c) mg*

**Explanation:** $v = u + \frac{1}{2} \times gt^2$

Differentiate $y$ with respect to $t$ which gives velocity as

$$\frac{dy}{dt} = v$$

$$\frac{dv}{dt} = u + \frac{1}{2}g(2t)$$

$$v = u + gt \quad \text{...(i)}$$

Differentiate $v$ with respect to $t$ which gives velocity as

$$\frac{d^2y}{dt^2} = \frac{dv}{dt} = a$$

$$\frac{dv}{dt} = 0 + g$$

$$a = g \quad \text{...(ii)}$$

We know that force acting on a mass $m$ is given by $F = ma$

Now from Eq. (ii), substitute $a = g$.

Hence, $F = mg$

(E) *(b) 10 N*

**Explanation:** Here total mass of the rope, m = 10 × 0.5 = 5 kg

Acceleration of the rope, $a = \frac{25}{5} = 5 \text{ m/s}^2$

The mass of remaining rope 10 – 6 = 4 m is $m_r = 4 \times 0.5 = 2$ kg

The tension will be

$$T = m_r a$$
$$= 5 \times 2 = 10 \text{ N}$$

## VERY SHORT ANSWER Type Questions (VSA)

[ 1 mark ]

**33. Explain why the water doesn't fall even at the top of the circle when the bucket full of water is upside down rotating in a vertical circle? [Delhi Gov. QB 2022]**

**Ans.** The weight of the water and bucket is expended in creating the required centripetal force at the circle's apex.

**34. When two surfaces in contact are polished beyond a certain point, why does frictional force increase?**

**Ans.** When two surfaces in contact are polished beyond a certain limit and made extremely smooth, they come in intimate contact with each other. Now, the force of adhesion comes into play. Due to this force, the motion of one surface over another surface becomes retarding and hence causes an increase in the friction.

**35. A force is being applied to a body but it causes no acceleration. What possibilities may be considered to explain the observation? [Diksha]**

**Ans.** If the force is a deforming force, then it does not produce the acceleration and the force is an internal force that cannot cause acceleration, these two properties, when applied on a body causes no acceleration.

**36. You accelerate your car forward. What is the direction of the frictional force on a package resting on the floor of the car? [Delhi Gov. QB 2022]**

**Ans.** The package in the accelerating automobile (a non-inertial frame) receives a Pseudo force in the opposite direction of the car's velocity. The frictional force on the package that operates in the opposite direction (forward) as the automobile is therefore in the same direction (forward).

**37. Mention the factor on which the coefficient of friction depends? Coefficient of friction depends on the nature of surfaces and their roughness that are in contact.**

**The road at a turn of radius 24 m is inclined by an angle of 9°. With what speed, a car should move on the turn so the normal contact force is able to provide the necessary centripetal force? [Diksha]**

**Ans.** If $v$ is the correct speed, then,

$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta}$$

$$= \sqrt{24 \times 9.8 \times \tan 9^\circ}$$

$$= 6.1 \text{ m/s}$$

**38. On a rainy day skidding takes place along a curved path. Why? [Delhi Gov. QB 2022]**

**Ans.** On a wet day, the friction between the tyres and the road decreases.

**39. A person driving a car suddenly applies the brakes on seeing a child on the road ahead. If he is not wearing a seat belt, he falls forward and hits his head against the steering wheel. Why? [NCERT Exemplar]**

**Ans.** When a person driving a car suddenly applies the brakes, the lower part of the body slows down with the car while the upper part of the body continues to move forward due to inertia of motion. If the driver is not wearing a seat belt, then he falls forward and his head hits against the steering wheel.

**40. Lubricants are used between the two parts of a machine. Why? [Delhi Gov. QB 2022]**

**Ans.** To decrease friction, and thereby wear and tear.

**41. Chinawares are wrapped in straw paper before packing. Why? [Delhi Gov. QB 2022]**

**Ans.** Chinaware is covered in straw or paper before packaging so that if it falls, the impact will take longer to reach. The chinaware is supported by a straw of paper, thus the average force exerted on the chinaware is tiny, and the possibilities of it shattering are reduced.

**42. Bodies of larger mass need greater initial effort to put them in motion. Why? [Delhi Gov. QB 2022]**

**Ans.** As we know that F = $ma$, a greater force will be required to move a big mass.

# SHORT ANSWER Type-I Questions (SA-I)

[ 2 mark ]

**43. Two forces $(4\hat{i}+10\hat{j})$ N and $(8\hat{i}+8\hat{j})$ N are acting on the body of mass 6 kg. Then find the acceleration that develops in body.**

**Ans.** Given that: $\vec{F_1}=(4\hat{i}+10\hat{j})$N

and $\vec{F_2}=(8\hat{i}+8\hat{j})$N

Net force on the body of mass 6 kg.

$$\vec{F}=\vec{F_1}=\vec{F_2}$$

$$=(4\hat{i}+10\hat{j})+(8\hat{i}+8\hat{j})$$

$$=12\hat{i}+18\hat{j}$$

Acceleration of the body,

$$\vec{a}=\frac{\vec{F}}{m}=\frac{12\hat{i}+18\hat{j}}{6}$$

$$=\frac{(2\hat{i}+3\hat{j})m}{s^2}$$

Magnitude of the acceleration,

$$a=|\vec{a}|=\sqrt{2^2+3^2}$$

$$=\sqrt{4+9}=\sqrt{13}\ \text{m/s}^2$$

**44. A uniform rope of mass *m* hangs freely from a ceiling. A monkey of mass M climbs up the rope with an acceleration *a*. Find the force exerted by the rope on the ceiling. [Diksha]**

**Ans.** The force on the ceiling is equal to the sum of weight of rope and force applied by monkey on the rope.

Force applied by monkey on the rope,

$$F = M(g + a)$$

and the weight of the rope = $mg$

Therefore,

Force on the ceiling = $mg$ + M $(g + a)$

**45. A bob of mass 0.1 kg hung from the ceiling of room by a string 2 m long is oscillation. At its mean position the speed of a bob is 1 ms$^{-1}$. What is the trajectory of the oscillating bob if the string is cut when the bob is:**

**(A) At the mean position**

**(B) At its extreme position**

**[Delhi Gov. QB 2022]**

**Ans.** (A) The bob's speed is zero at its most extreme position. If the string is severed, it will tumble down vertically.

(B) The bob has a horizontal velocity at the mean location. If the thread is severed, it will follow a parabolic route.

**46. Two blocks made of different metals identical in shape and size are acted upon by equal forces, which cause them to slide on a horizontal surface. The acceleration of the second block is found to be 5 times that of the first, what is the ratio of the mass of the second to first?**

**Ans.** Let F be resultant force on each block.

For 1st block, $F_1 = m_1a_1$

For 2nd block, $F_2 = m_2a_2$

or, $$\frac{m_1a_1}{m_2a_2}=1$$

i.e., $$\frac{m_1}{m_2}=\frac{a_2}{a_1} \quad \text{... (i)}$$

Here, $a_2 = 5a_1$ ... (ii)

From (i) and (ii),

$$m_1 : m_2 = 5 : 1$$

**47. State Newton's second, law of motion. Express it mathematically and hence obtain a relation between force and acceleration.**

**Ans.** According to Newton's second law, the rate of change of momentum is directly proportional to the force.

i.e., $\vec{F}$ = rate of change of momentum = $\left(\frac{d\vec{p}}{dt}\right)$

$$\vec{F}=k\frac{d\vec{p}}{dt}$$

$$\vec{p}=m\vec{v}$$

$$\vec{F}=km\frac{d\vec{v}}{dt}$$

$$\vec{F}=km\vec{a}$$

(In S.I. unit $k = 1$)

**48. Calculate the magnitude and direction of the net force that acts on**

**(A) A drop of rain falling with constant speed.**

**(B) A kite held stationary in the sky.**

**Ans.** (A) According to the first law of motion, if a particle moves with constant speed, then $a = 0$. Therefore, F = 0

(B) As the kite is stationary, so net force on the kite will also be zero.

**49.** A woman throws an object of mass 500 g with a speed of 25 m/s:

(A) What is the impulse imparted to the object?

(B) If the object hits a wall and rebounds with half the original speed, what is the change in momentum of the objects?

[Delhi Gov. SQP 2022]

**Ans.** Here, $m = 500$ g $= \frac{1}{2}$ kg, $u = 25$ ms$^{-1}$

(A) Impulse imparted = change in momentum

= initial momentum given $mu = \frac{1}{2} \times 25$

$= 12.5$ Ns

(B) On rebounding force the wall

$$v = \frac{1}{2} \times 25$$

$$= -12.5 \text{ m/s}$$

Change in momentum $= m(v - u)$

$$= \frac{1}{2}(-12.5 - 25)$$

$$= -18.75 \text{ kgms}^{-1}$$

**50.** A cycle moves in a circular motion of radius 30 cm at a speed that increases uniformly. If the speed changes from 4.0 m/s to 7.0 m/s in 2.0 s, find the angular acceleration. [Diksha]

**Ans.** The tangential acceleration is,

$$a_t = \frac{dv}{dt}$$

$$= \frac{v_2 - v_1}{t_2 - t_1}$$

$$= \frac{7-4}{2} = 1.5 \text{ m/s}^2$$

The angular acceleration is,

$$\alpha = \frac{a_t}{r}$$

$$= \frac{1.5 \text{ m/s}^2}{30 \text{ cm}} = 5 \text{ rad/s}^2$$

**51.** A block is kept on a polished plank of the same material. The inclination of the plank is increased slowly. It is observed that the block starts slipping down when the plank makes an angle of 20° with the horizontal axis. However, if it started, then the block will continue with uniform speed if the inclination is reduced to 17°. Find the coefficients of static and kinetic friction between the block and the plank. [Diksha]

**Ans.** The coefficient of static friction is,

$$\mu_s = \tan 20°$$

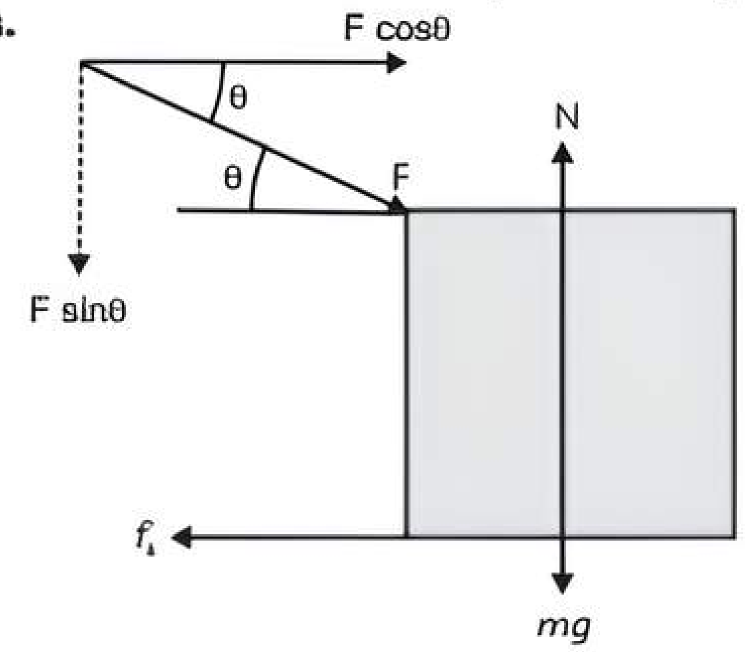
and the coefficient of kinetic friction is,

$$\mu_k = \tan 17°$$

**52.** It is easier to pull a roller than to push it. Why? (using vector diagram)

[Delhi Gov. QB 2022]

**Ans.**




A body's motion is determined by the effective force applied to it, which includes its weight. Because the effective weight of pulling is smaller than that of pushing, pulling requires less force. As a result, pulling is easier than pushing.

## SHORT ANSWER Type-II Questions (SA-II)

[ 3 marks ]

**53.** A rocket with a lift-off mass 20,000 kg is blasted upwards with an initial acceleration of 5.0 ms$^{-2}$. Calculate the initial thrust (force) of the blast.

**Ans.** Mass of the rocket, $m = 20{,}000$ kg

Initial acceleration, $a = 5$ m/s$^2$

Acceleration due to gravity, $g = 10$ m/s$^2$

Using Newton's second law of motion, the net force (thrust) acting on the rocket is given by the relation:

$$F - mg = ma$$

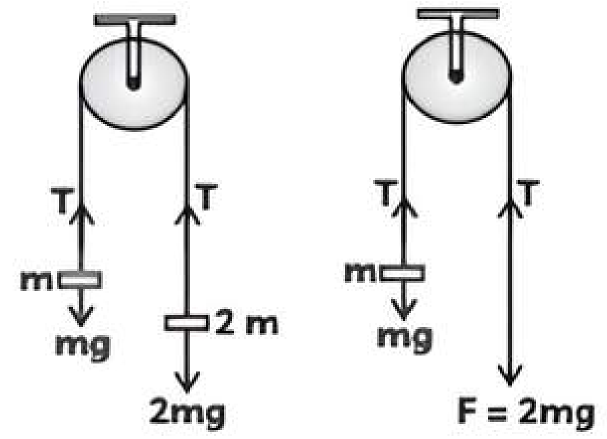
$$F = m(g + a)$$

$$= 20000 \times (10 + 5)$$

$$= 20000 \times 15$$

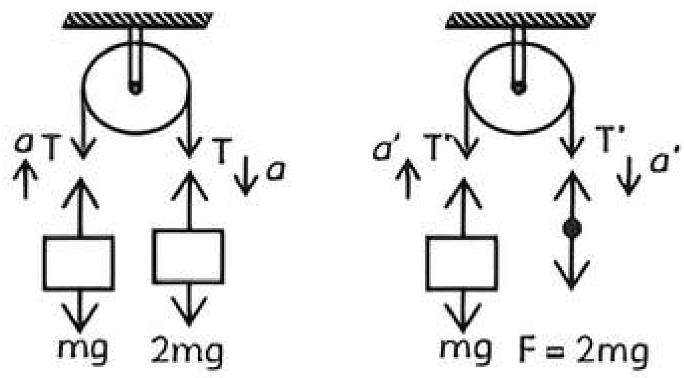
$$= 3 \times 10^5 \text{ N}$$

**54.** The pulley arrangement of fig. are identical. The mass of the rope is negligible. In (*a*) mass *m* is lifted up by attaching a mass (2 *m*) to the other end of the rope. In (*b*), *m* is lifted up by pulling the other end of the rope with a constant downward force F = 2 mg. In which case, the acceleration of $m$ is more?




[Delhi Gov. QB 2022]

**Ans.**




Let the accelerations in both cases are $a$ and $a'$ respectively.

The tension is given as,

$$T - mg = ma \quad \text{...(i)}$$

$$2\,mg - T = 2\,ma \quad \text{...(ii)}$$

By adding equations (i) and (ii), we get

$$mg = 3\,ma$$

$$a = \frac{g}{3}$$

In the second case the tension is given as,

$$T' - mg = ma' \quad \text{...(iii)}$$

$$2\,mg - T' = 0 \quad \text{...(iv)}$$

By adding equation (iii) and (iv), we get

$$mg = ma'$$

$$a' = g$$

The ratio of acceleration is given as,

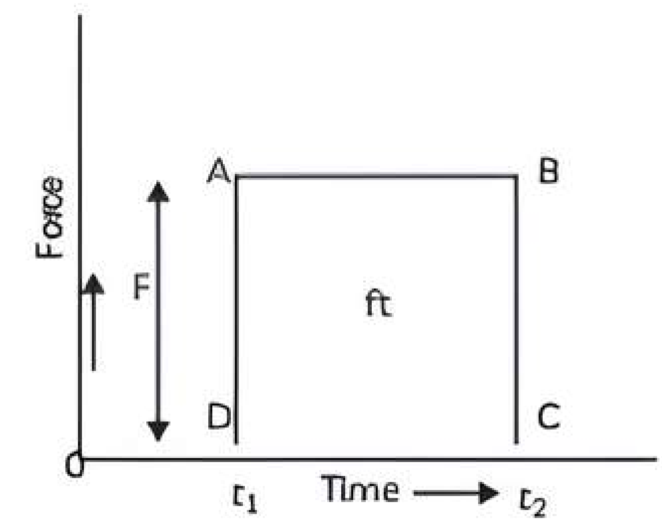
$$\frac{a}{a'} = \frac{\frac{g}{3}}{g}$$

$$\frac{a}{a'} = \frac{1}{3}$$

**55.** **Suppose If you jump to the ground from any height, you bend your knees upon impact, extending the time of collision and lessening the impact force. It is the common situation which depicts the impulse force which explains that if an impact stops a moving object, then the change in momentum is a fixed quantity, and extending the time of the collision will decrease the time average of the impact force by the same factor. Deduce how this force works on a constant and variable body.**

**Ans.** The impulse of force can be extracted and found to be equal to the change in momentum of an object, provided the mass is constant.

When a constant force acts on a body: Suppose a constant force F acts on a body from time $t_1$ to $t_2$ the force-time graph is a straight line AB parallel to the time-axis.

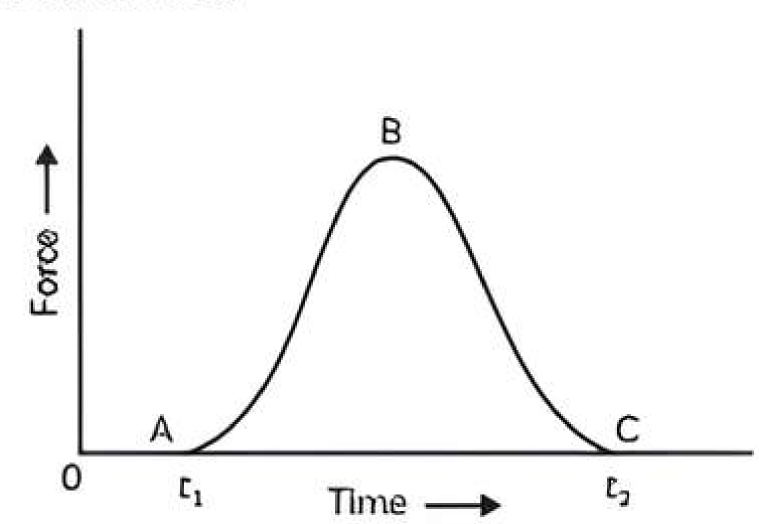



Area of rectangle ABCD = AD × AB

$$= F\,(t_2 - t_1) = F \times t$$

= Magnitude of the impulse of force F in time interval $t$

**When a variable force acts on the body:** Suppose a force varying in magnitude acts on a body for time $t_2 - t_1 = t$. The force-time graph is a curve ABC.




Impulse of force F in time interval $t_1$,

$$I = \int_0^t F\,dt = \text{Area under the force} - \text{time curve ABC}$$

Thus, the area under the force-time graph gives the magnitude of the impulse of the given force in the given time interval.

**56.** **A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 m/s. How long does the body take to stop?** **[Diksha]**

**Ans.** Retarding force, F = – 50 N

Mass of the body, $m$ = 20 kg

Initial velocity of the body, $u$ = 15 m/s

Final velocity of the body, $v$ = 0

Using Newton's second law of motion, the acceleration ($a$) produced in the body can be calculated as:

$$F = ma$$

$$-50 = 20 \times a$$

Therefore, $$a = \frac{-50}{20}$$

$$= -2.5 \text{ m/s}^2$$

Using the first equation of motion, the time (t) taken by the body to come to rest can be calculated as:

$$t = \frac{-u}{a}$$

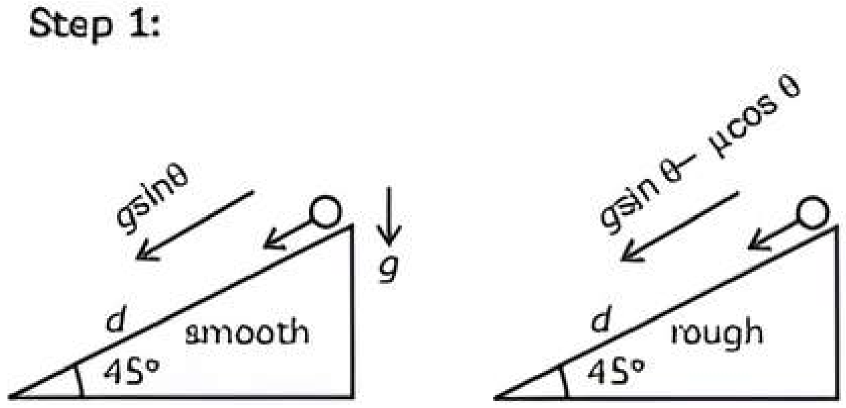
$$= -\frac{15}{-2.5} = 6 \text{ s}$$

**57. Smooth block is released at rest on a 45° incline and then slides a distance *d*. If the time taken of slide on rough incline is *n* times as large as that to slide than on a smooth incline. Show that coeffcient of friction.**

$$\mu = \left(1 - \frac{1}{n^2}\right)$$

**Ans. Hint:** Draw free body diagram and use equation motion.

**Step 1:**




**Step 2:** Distance travelled on both surfaces.

$$d = \frac{1}{2} g\sin\theta \times t_1$$

$$d = \frac{1}{2} g(\sin\theta - \mu\cos\theta) \cdot t_2^2$$

$$\therefore t_1 = \sqrt{\frac{2d}{g\sin\theta}}; \quad t_2 = \sqrt{\frac{2d}{g(\sin\theta - \cos\theta)}}$$

$$\therefore t_2 = n\,t_1$$

$$\Rightarrow \sqrt[n]{\frac{2d}{g\sin\theta}} = \sqrt{\frac{2d}{g\sin\theta - \mu\cos\theta}}$$

$$\frac{n^2 \times 2d}{g\sin\theta} = \frac{2d}{g(\sin\theta - \mu\cos\theta)}$$

$$\Rightarrow \frac{n^2}{\sin 45^\circ} = \frac{1}{\sin 45^\circ - \mu\cos 45^\circ}$$

$$\mu_k = 1 - \frac{1}{n^2}$$

**58. A man weighs 196 N on a spring balance at the south pole. What will be its weight of that same man on the same scale if he weighs on the equator?**

**Use, $g = \frac{GM}{R^2}$ = 9.8 m/s$^2$and the radius of the earth (R = 6400 km.)**

**Ans.** At poles, the apparent weight is same as the true weight. Thus,

$$196 \text{ N} = mg = m \times (9.8 \text{ m/s}^2)$$

$$\therefore \quad m = 20 \text{ kg.}$$

At the equator, the apparent weight is,

$$mg = mg - m\omega^2 R.$$

As given, radius of the earth is 6400 km and the angular speed is

$$\omega = \frac{2\pi rad}{24 \times 60 \times 60 \sec}$$

$$= 7.27 \times 10^{-5} \text{ rad/sec}$$

Thus,

$$mg = 196 \text{ N} (20 \text{ kg}) (7.27 \times 10^{-5}) \times 2 \times (6400 \text{ km})$$

$$= 195.998 \text{ N} \approx 196 \text{ N}$$

**59.**

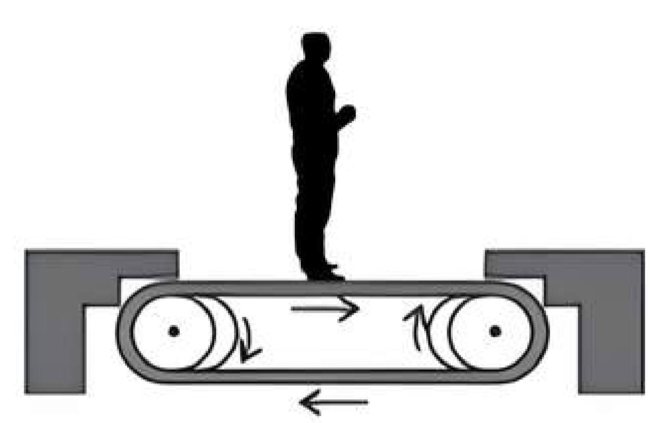


**Figure shows a man standing stationary with respect to a horizontal conveyor belt that is accelerating with. What is the net force on the man? If the coefficient of static friction between the man's shoes and the belt is 0.2, up to what acceleration of the belt can the man continue to be stationary relative to the belt? (Mass of the man = 65 kg.)**

**Ans.** Mass of the man, $m = 65$ kg

Acceleration of the belt, $a = 1\ m/s^2$

Coefficient of static friction, $\mu = 0.2$

The net force F, acting on the man is given by Newton's second law of motion as:

$$F'_{net} = ma$$

$$= 65 \times 1 = 65\ N$$

The man will continue to be stationary with respect to the conveyor belt until the net force on the man is less than or equal to the frictional force, exerted by the belt, i.e.,

$$F'_{net} = F_s$$

$$ma' = \mu mg$$

$$a' = 0.2 \times 10 = 2\ ms^{-2}$$

Therefore, the maximum acceleration of the belt up to which the man can stand stationary is 2 $ms^{-2}$.

**60. A fighter plane is pulling out for a dive at a speed of 900 km/hr. Assuming its path to be a vertical circle of radius 2000 m and its mass to be 16000 kg, find the force exerted by the air on it at the lowest point.**

**Take $g = 9.8\ m/s^2$.**

**Ans.** At the lowest point in the path the acceleration is vertically upward (towards the center) and its magnitude is 9.8 $m/s^2$. The forces on the plane are:

(1) weight $Mg$ downward and

(2) force F by the air upward.

Hence, Newton's second law of motion gives

$$F - Mg = \frac{Mv^2}{r}$$

$$v = 900\ km\ hr^{-1}$$

$$= \frac{9 \times 10^5}{3600} ms^{-1}$$

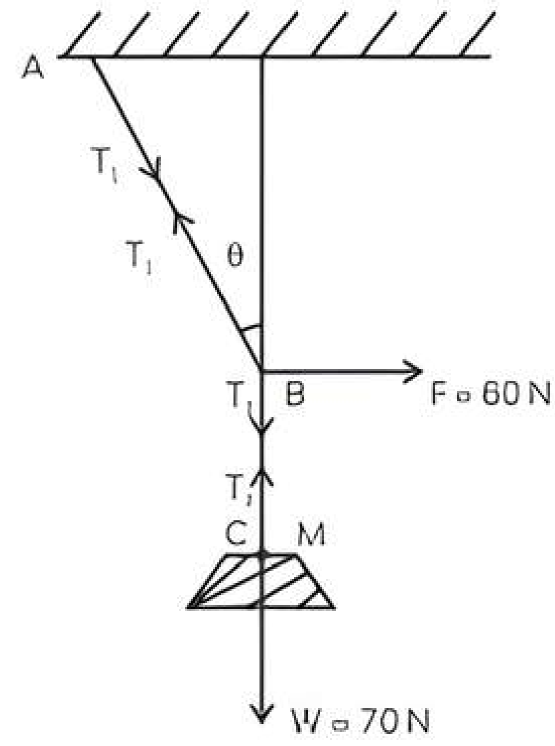
$$= 250\ m\ sec^{-1}$$

Here,

$$F = 16000\left(9.8 + \frac{62500}{2000}\right) N$$

$$= 6.56 \times 10^5\ N \quad \text{(upwards)}$$

**61. A weight of 7 kg is suspended by a 2 m long rope of negligible weight from ceiling. A horizontal force of 60 N applied at the mid-point of the rope. Find the angle of the rope makes equilibrium in vertical.**




**[Diksha]**

**Ans.** The arrange for the above situation as shown below:

The rope represents as ABC

where AB = BC = 1 m. Force (F = 60 N) represents a horizontal force which applies at point B while M is the suspended weight whose weight is W($mg$ = 70 N).

The tension $T_1$ and $T_2$ in two parts of the rope are as shown:

For equilibrium of the mass M,

we have,

$$T_2 = W = 70\ N$$

Again, for equilibrium of point B, we have

$$T_1 \cos\theta = T_2 = 70\ N$$

$$T_1 \sin\theta = F = 60\ N$$

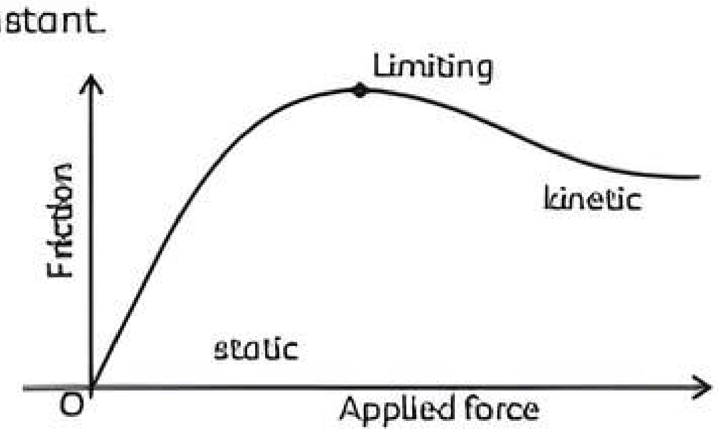
$$\tan\theta = \frac{6}{7} = 0.85$$

$$\theta = \tan^{-1}(0.85) = 40.4°$$

**62. With the help of suitable example, explain the terms static friction, limiting friction and kinetic friction. Show that static friction is a self-adjusting force. Also plot the graph showing the variation between applied force F and force of friction $f$. [Delhi Gov. QB 2022]**

**Ans.** Static friction is friction that acts on a body that is at rest. The greatest value of static friction is referred to as limiting friction. It is the amount of force necessary for the body to begin moving. Kinetic friction is the frictional force that acts on a moving body.

When the applied force is increased, static friction increases until it hits limiting friction, which is fixed, while kinetic friction remains constant.



# LONG ANSWER Type Questions (LA)

[ 4 & 5 marks ]

**63.** **A hunter has a maching gun that can fire 50 g bullets with a velocity of 150 ms$^{-1}$. A 60 kg tiger springs at him with a velocity of 10 ms$^{-1}$. How many bullets must the hunter fire into the target so as to stop him in his track?** **[Delhi Gov. QB 2022]**

**Ans.** Given $m$ = mass of bullet = 50 g = 0.50 kg

M = mass of tiger = 60 kg

$v$ = velocity of bullet – 150 m/s

V = velocity of tiger = – 10 m/s

($v$ it is coming from opposite direction $n$ = no. of bullets fired per second at the tiger so as to stop it.)

$$P_i = 0, \text{ before firing} \quad \text{...(i)}$$

$$P_f = n(mv) + MV \quad \text{...(ii)}$$

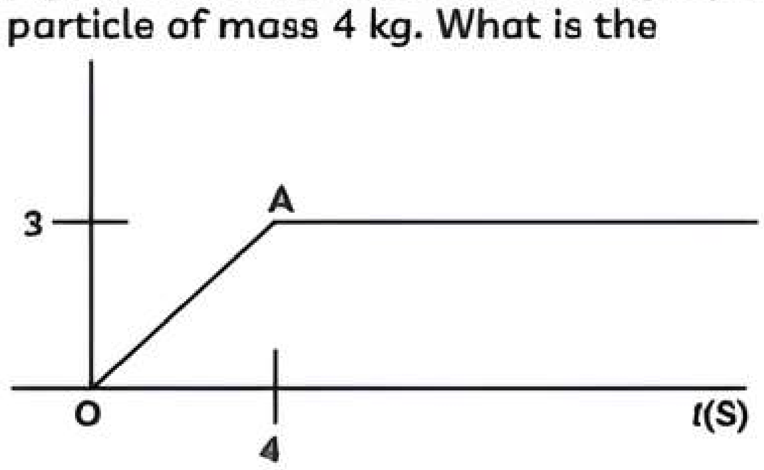
∴ From the law of conservation of momentum,

$$P_i = P_f$$

$$\Rightarrow \quad 0 = n(mv) + MV$$

$$n = \frac{MV}{mv} = \frac{-60\times(-10)}{0.05\times 150} = 80$$

**64.** **Fig. shows the position-time graph of a particle of mass 4 kg. What is the**




**(A) force on the particle for $t < 0$, $t > 4s$, $0 < t < 4s$?**

**(B) impulse at $t = 0$ and $t = 4s$? (Consider one dimensional motion only)** **[Delhi Gov. QB 2022]**

**Ans.** (A) From the given graph, for $t < 0$, and $t > 4s$, the position of particle does not change.

Therefore, the particle is at rest and for $0 < t < 4s$ the position-time graph is a straight line.

This implies the velocity of the particle is constant.

Therefore,

Acceleration of particle for $t < 0$, $t < 4s$ and $0 < t < 4s$ is zero. Hence, no force acts on the particle.

(B) We know, the slope of the position-time graph is velocity. Therefore, velocity of particle in interval $0 < t < 4s$ is,

$$v = \frac{x(4) - x(0)}{4 - 0}$$

$$= \frac{3-0}{4-0} = \frac{3}{4} \text{ m/s}$$

Momentum of particle at $t < 0$ is,

$$p_1 = m \times 0 = 0$$

Momentum of particle at $t > 0$ is,

$$p_2 = 4 \times \left(\frac{3}{4}\right) = 3 \text{ Ns}$$

Therefore,

Impulse at $t = 0$ is,

$$I = p_2 - p_1 = 3 \text{ Ns}$$

Now, momentum of particle at time $t < 4s$ is,

$$p_3 = 3 \text{ Ns}$$

Momentum of particle at $t < 4s$ is,

$$p_4 = m \times 0 = 0$$

Therefore, impulse at $t$ = 4 sec is,

$$I = (p_4 - p_3) = -3 \text{ Ns}$$

**65.** **An auto is running with 40 km/h speed then auto driver sees an old man sitting in the middle of a highway and he takes himself in the position of rest in 5 sec to save an old man. Find out the value of average retarding force on auto? Also calculate the distance between auto and old man at the time when auto driver applied brakes. The weight of an auto and driver is 400 kg and 60 kg respectively.**

**Ans.** The initial speed of an auto is $u$ = 40 km/h

$$u = \frac{40\times 1000}{60\times 60} = 11.11 \text{ m/s}$$

An auto driver takes his auto in rest position in 5 sec.

So, from first equation of motion, $v = u + at$,

the retardation produced by applying brakes is:

$$a = \frac{v-u}{t}$$

$$= \frac{0-11.11}{5s} \text{ms}^{-1}$$

$$= -2.22 \text{ ms}^{-2}$$

The total weight of the vehicle and the driver is,

$$m = 400 \text{ kg} + 60 \text{ kg}$$

$$= 460 \text{ kg}$$

By applying newton's second law, the deceleration force applied on an auto is:

$$F = ma$$
$$= 460 \text{ kg} \times (-2.22 \text{ ms}^{-2})$$
$$= -1021.2 \text{ N}$$

Assume that the old man was at a distance $s$ from an auto when brakes were applied then from eq., then we have,

$$s = \frac{v^2 - u^2}{2a}$$
$$= \frac{0-(11.11 \text{ ms}^{-1})^2}{2(-2.22 \text{ ms}^{-2})}$$
$$= 27.8 \text{ m}$$

**66. A 300 kg truck runs with a uniform speed of 72 km/h on a frictionless track. A weight of 20 kg person on truck from one end to other end of truck *i.e.*, 15 m away with velocity of 5 $ms^{-1}$ in opposite direction of truck's motion and jumps out of the truck. Calculate the final velocity of the truck? How much distance is covered by truck from the time the person starts to run? [Diksha]**

**Ans.** Let M = 300 kg be the weight of the truck and $m$ = 20 kg of the person.

The initial speed of the truck is:

$$u = \frac{72 \text{ km}}{h} = \frac{72 \times 1000}{60 \times 60} = 20 \text{ ms}^{-1}$$

Before the person starts running, the (starting) linear momentum of (truck + person) is:

$$p_1 = (M + m)u$$
$$= (300 + 20) \times 20$$
$$= 6400 \text{ kgms}^{-1}$$

When the person runs with the speed of 5 $ms^{-1}$ which is relative to the truck in opposite direction of truck's motion is $(v - 5)$ $ms^{-1}$

Therefore, the linear momentum of (truck + person), when the person is running

$$p_2 = Mv = u(v - 5 \text{ ms}^{-1})$$
$$= (300 \text{ kg})v + (20 \text{ kg})(v - 5 \text{ ms}^{-1})$$
$$= (320 \text{ kg})v - 1100 \text{ kg ms}^{-1}$$

As, no external force is applied on the system, by using conservation of linear momentum,

$$p_1 = p_2$$
$$6400 \text{ kg ms}^{-1} = (300 \text{ kg})v - 100 \text{ kg ms}^{-1}$$
$$v = \frac{6500 \text{ kgms}^{-1}}{320 \text{ kg}}$$
$$= 20.35 \text{ ms}^{-1}$$

The time taken by the child in running a distance of 15 m on the truck with velocity 5 which is relative to the truck (before the person jumps) is:

$$t = \frac{15 \text{ m}}{5 \text{ ms}^{-1}} = 3 \text{s}$$

Hence, the distance moved by the truck during this time

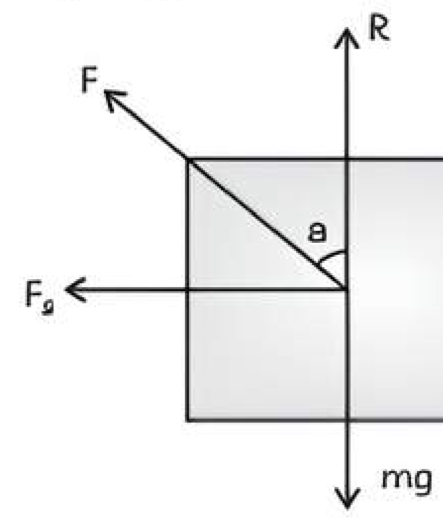
$$v \times t = 20.35 \text{ ms}^{-1} \times 3$$
$$= 60.9 \text{ m}$$

**67. Describe the relation of coefficient of friction and angle of friction and the reaction.**

**Angle of friction is the angle between the resultant of frictional force and normal reaction, R. $\tan\alpha = \frac{F_s}{R}$, with the normal reaction.**

**Ans.** Coefficient of friction is the ratio of the static frictional force and normal reaction.

$$F_s \propto R$$
$$F_s = \mu_s R$$




$$\therefore \quad \mu_s = \frac{F_s}{R}$$

From above equations,

We can say that

$$\tan\alpha = \mu_s = \frac{F_s}{R}.$$

**68. The coefficient of static friction between a block of mass $m$ and an incline is $\mu_s = 0.3$.**

**(A) What can be the maximum angle $\theta$ of the incline with the horizontal so that the block does not slip on the plane?**

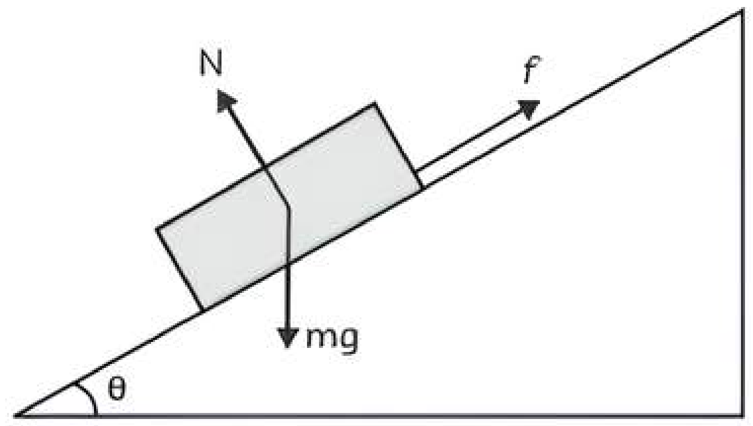
**(B) If the incline makes an angle $\frac{\theta}{2}$ with the horizontal, find the frictional force on the block. [Diksha]**

**Ans.** (A) The forces on the block are:

(1) The weight $mg$ downward by the earth.

(2) The normal contact force N by the incline, and

(3) The friction F parallel to the incline up the plane, by the incline.




As the block is at rest, these forces should add up to zero. Also, since θ is the maximum angle to prevent slipping, this is *a* case of limiting equilibrium and so,

$$F = \mu_s N$$

Taking components perpendicular to the incline,

$$N - mg \cos\theta = 0$$

$$N = mg \cos\theta \quad ...(i)$$

Taking components parallel to the incline,

$$F - mg \sin\theta = 0$$

or, $$F = mg \sin\theta$$

or, $$\mu_s N = mg \sin\theta \quad ...(ii)$$

Dividing eqn. (ii) by (i),

$$\mu_s = \tan\theta$$

as given, $$\mu_s = 0.3,$$

∴ $$\tan\theta = (0.3)$$

(B) If the angle of inclination reduced to $\frac{\theta}{2}$, so,

the equilibrium is not limiting, and hence the force of static

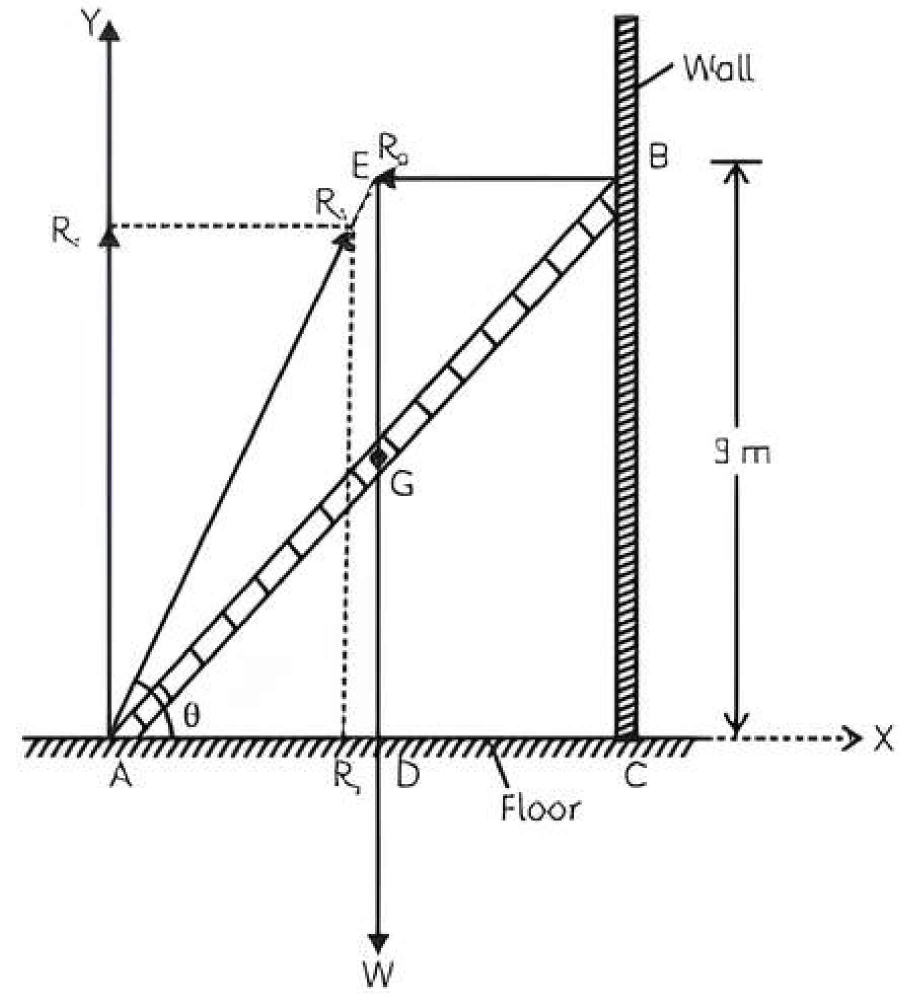
Friction F is less than $\mu_s N$. To know the value of *f*, we proceed as in part (A) and get the equations

$$N = mg \cos\frac{\theta}{2}$$

$$F = mg \sin\frac{\theta}{2}$$

Thus, the force of friction will be $mg \sin\frac{\theta}{2}$.

**69.** A homogeneous ladder AB of weight 50 kg which is 5 m long, rests against a bumpy horizontal floor and smooth vertical wall. The wall having height through which the ladder makes the contact is 3 m from the floor. Find out the reaction at this point and the contact force with its component at the floor?




**Ans.** Let the reaction at point A (Point of contact of ladder and floor) makes an angle with the horizontal.

Resolving along X and Y axis, we have

$$R_x = R_A \cos\theta \quad ...(i)$$

$$R_y = R_A \sin\theta \quad ...(ii)$$

and since, AB = 5 m, BC = 3 m (given)

Therefore, AC = 4 m

Now, since the wall is smooth therefore, reaction is horizontal normal to the wall. Further reaction $R_A$, weight W and are concurrent, *i.e.*, they are in contact with point B therefore, the ladder is in the state of equillibrium.

Considering translational equilibrium,

$$\Sigma F_x = 0$$

and $$\Sigma F_y = 0$$

We have,

$$R_x - R_B = 0$$

which gives,

$$R_x = R_B \quad ...(iii)$$

And $$R_y - W = 0, \text{ gives,}$$

$$R_y = W \quad ...(iv)$$

Now considering rotational equillibrium $\Sigma_p = 0$

Take moments about a point at which maximum number of forces are acting (this may save some valuable time in calculation part). For convenience let us take point A.

$$-(W \times AD) + (R_B \times BC) = 0$$

By convention clockwise momentum is negative and an anticlockwise momentum is positive or

$$(-50 \times 2) + (R_B \times 3) = 0$$

$$R_B = \frac{50 \times 2}{3}$$

$$= 33.33 \text{ kg.wt}$$

Now from eqn. (iii) and (iv),

$$R_x = 33.33 \text{ kg.wt}$$

$$R_y = 50 \text{ kg.wt}$$

Now squaring and adding eqn. (i) and (ii), we have

$$R_x^2 + R_y^2 = R_A^2 \sin^2\theta + R_A^2 \cos^2\theta$$

$$R_A = \sqrt{R_x^2 + R_y^2}$$

$$R_A = \sqrt{33.33^2 + 50^2}$$

$$R_A = 60.09 \text{ kg.wt}$$

And the direction of ladder can be obtained by dividing eqn. (ii) by eqn. (i)

$$\frac{R_A \sin\theta}{R_A \cos\theta} = \frac{R_y}{R_x}$$

$$\Rightarrow \quad \tan\theta = \frac{R_y}{R_x}$$

Or, $$\theta = \tan^{-1}\theta = \frac{R_y}{R_x}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{50}{33.33}\right)$$

$$\theta = \tan^{-1}(1.5)$$

$$\theta = 56.31°$$

**70. For a collision occurring between object 1 and object 2 in an isolated system, the total momentum of the two objects before the collision is equal to the total momentum of the two objects after the collision. That is, the momentum lost by object 1 is equal to the momentum gained by object 2. If two particles collide with each other, then prove that Newton's second and third law will help in conservation of momentum.**

**Ans.** If no external force acts on a system (called isolated) of constant mass, the total momentum of the system remains constant with time.

According to this law for a system of particles,

$$\vec{F} = \frac{d\vec{p}}{dt}$$

In absence of external force, $\vec{F} = 0$.

then $\vec{P}$ = constant.

i.e., $\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots = \text{constant}$

or, $m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots = \text{constant}$

This equation shows that, in the absence of external force for a closed system, the linear momentum of individual particles may change, but their sum remains unchanged with time.

Conservation of linear momentum is equivalent to Newton's third law of motion.

For a system of two particles in absence of external force by law of conservation of linear momentum.

$$\vec{p}_1 + \vec{p}_2 = \text{constant}$$

Therefore, $m_1\vec{v}_1 + m_2\vec{v}_2 = \text{constant}$

Differentiating above with respect to time,

$$m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} = 0$$

$$\Rightarrow \quad m_1\vec{a}_1 + m_2\vec{a}_2 = 0a$$

$$\Rightarrow \quad \vec{F}_1 + \vec{F}_2 = 0$$

Therefore, $\vec{F}_2 = -\vec{F}_1$

i.e., for every action there is an equal and opposite reaction, which is Newton's third law of motion.

In case of collision between particles, equal and opposite forces will act on individual particles by Newton's third law.

Hence, total force on the system will be zero.

### Related Theory

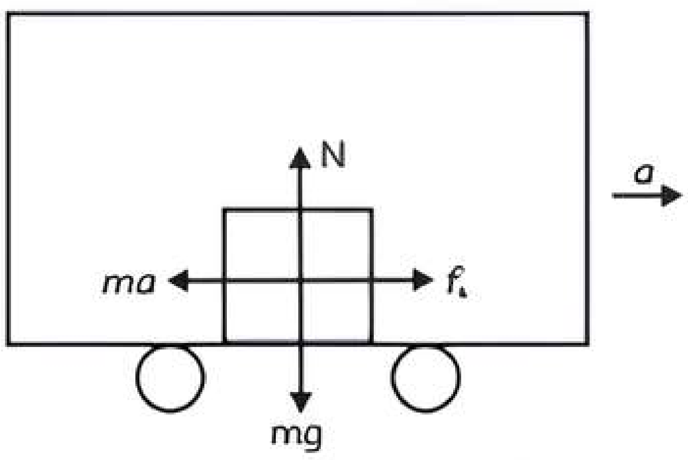
- *We should not confuse with systems and individual particles. As total force on the system of both particles is zero, but force acts on individual particles.*

*Law of conservation of linear momentum is independent of frame of reference, though linear momentum depends on frame of reference.*

## NUMERICAL Type Questions

**71.** **Determine the maximum acceleration of the train in which a box lying on its floor will remain stationary, given that the coefficient of static friction between the box and the train's floor is 0.15. (2m)**

**Ans.**




Given: $\mu_s = 0.13$; $g = 9.8\ m/s^2$; $a_{max} = ?$

The frictional force acting between the floor of the train and the surface of the box will oppose the slipping of the box on the floor of the train.

∴ Limiting friction force

$$f_s = ma_{max}$$

and $f_s = \mu_s N = \mu_s mg$

∴ $ma_{max} = \mu_s mg$

or $a_{max} = \mu_s g = 0.13 \times 9.8$

or $a_{max} = 1.274\ ms^{-2}$

or $a_{max} = 1.27\ ms^{-2}$

**72.** **A constant force acting on a body of mass 3.0 kg changes its speed from 2.0 m/s to 3.5 m/s in 25 s. The direction of the motion of the body remains unchanged. Calculate the magnitude and direction of the force. (2m)**

**Ans.** In the direction of motion of the body.

Mass of the body is given as, $m = 3$ kg

Initial speed of the body is given as, $u = 2\ ms^{-1}$

Final speed of the body is given as, $v = 3.5\ ms^{-1}$

Time is given as, $t = 25$ s

The acceleration ($a$) produced in the body can be estimated using the first equation of motion:

$$v = u + at$$

$$\therefore \quad a = \frac{v-u}{t}$$

$$= \frac{3.5-2}{25} = \frac{1.5}{25} = 0.06\ m/s$$

Newton's second law of motion states that force is equal to:

$$F = ma$$

$$= 3 \times 0.06 = 0.18\ N$$

The net force acting on the body is in the direction of its motion, since the application of force does not affect the direction of the body.

**73.** **An aircraft executes a horizontal loop at a speed of 720 km/hr with its wings banked at 15°. What is the radius of the loop? [Diksha](2m)**

**Ans.** From the question, it is given that the speed of the aircraft.

$$v = 720\ kmh^{-1}$$

$$= 720 \times \frac{5}{18}$$

$$= 200\ m/s$$

Also, the acceleration due to gravity, $g = 10\ m/s^2$ and the angle of banking, $\theta = 15°$

Therefore, for radius $r$, of the loop, we have the relation:

$$\tan\theta = \frac{v^2}{rg}$$

$$r = \frac{v^2}{g\tan\theta}$$

$$= \frac{200 \times 200}{10 \times \tan 15}$$

$$= \frac{4000}{0.268}$$

$$= 14925.37\ m$$

$$= 14.92\ km$$

**74.** **A girl riding a bicycle along a straight road with a speed of 5 $ms^{-1}$ throws a stone of mass 0.5 kg which has a speed of 15 $ms^{-1}$ with respect to the ground along her direction of motion. The mass of the girl and bicycle is 50 kg. Does the speed of the bicycle change after the stone is thrown? What is the change in speed, if so? [NCERT Exemplar] (2m)**

**Ans.** Yes, due to the principle of conservation of momentum, momentum will remain the same.

Initial momentum = $50.5 \times 5\ kgms^{-1}$

Final momentum = $(50\ v + 0.5 \times 15)\ kgms^{-1}$

Therefore, $50.5 \times 5 = 50\ v + 0.5 \times 15$

$$252.5 = 50\ v + 7.5$$

$$245 = 50\ v$$

$$v = 4.9\ m/s.$$

**75.** **A force of 5 N changes the velocity of a body from 10 m/s to 20 m/s in 5s. How much force is required to bring about the same change in 2s? [Diksha](2m)**

**Ans** Force = Mass × Acceleration

Acceleration = (Change in Vel./Time)

$$= \left(\frac{(20-10)}{5}\right)$$

$$= 2\ m/s^2$$

Therefore,

$$5 = \text{Mass} \times 2$$

$$\text{Mass} = 2.5 \text{ kg}$$

To bring the same change in 2 secs,

$$\text{Acceleration} = \left(\frac{(20-10)}{2}\right) = 5 \text{ m/s}^2$$

$$\text{Force} = 2.5 \times 5 = 12.5 \text{ N}$$

**76.** **A train runs along an unbanked circular bend of radius 30 m at a speed of 54 km/hr. The mass of the train is 106 kg. What provides the necessary centripetal force required for this purpose—The engine or the rails? What is the angle of banking required to prevent wearing out of the rail? (3m)**

**Ans.** From the question, we have the radius of circular bend given as, $r = 30\ m$

Speed of train, $v = 54$ km/hr

$= 54 \times 518 = 15$ m/s

Mass of train given as, $m$ =106 kg

Then we need to find the angle of banking θ.

(1) The centripetal force is generated by the lateral force exerted by rails on the train's wheels.

(2) The centripetal force is provided by the lateral thrust by the outer rail.

(3) According to Newton's third law of motion, the train exerts (*i.e.*, causes) an equal and opposite thrust on the outer rail causing its wear and tear.

Therefore, the angle of baking:

$$\tan\theta = \frac{v^2}{rg} = \frac{15^2}{30 \times 9.8}$$

$$\theta = \tan^{-1}(0.76)$$

$$\theta = 37.4$$

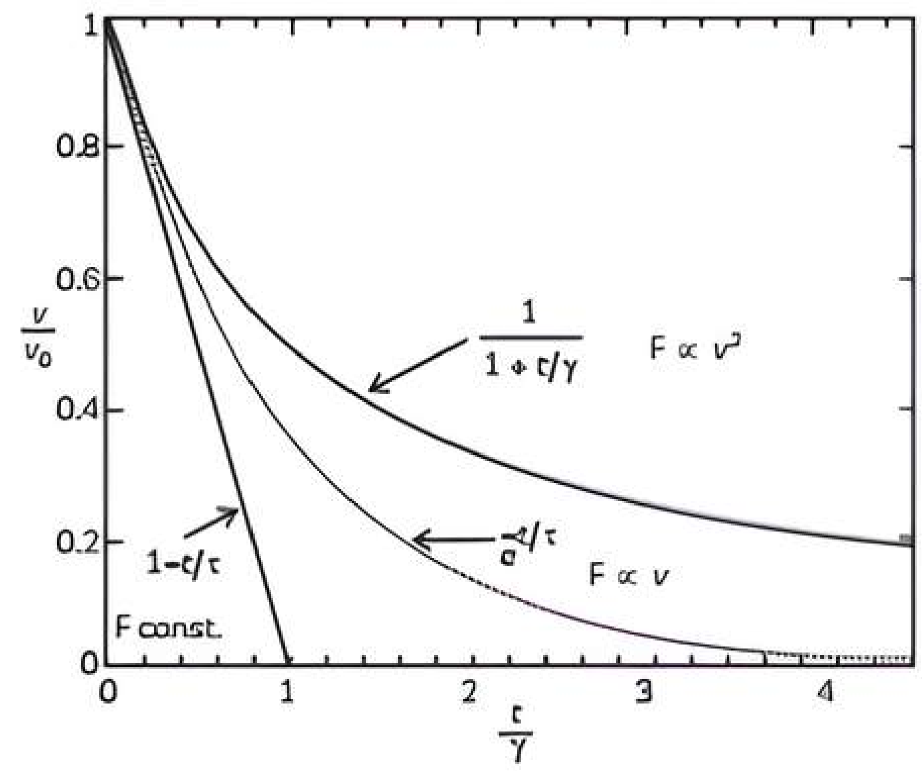
**77.** **Consider an object moving in one dimension projected with initial velocity $v_0$ (in the positive direction) at time $t = 0$. If no friction is present, then $v = v_0$ at all times $t$. If friction is present, the object will slow down. Three types of frictional forces will be considered:**

**(A) A constant frictional force ($b$) of the sort encountered when the object is sliding over a surface. The constant, $b$, has the units of N. $F = -b$**

**(B) A frictional drag force which varies linearly with the velocity. The constant, $b_1$, has the units of Ns/m or Kg/s. $F = -b_1v^2$**

**(C) A frictional drag force that varies quadratically with the velocity (as is often the case for higher velocity objects traveling in air). The constant, $b_2$, has the units of Kg/m. $F = -b_2v^2$ (5m)**

**Ans.** As can be seen from the calculations that follow (and the graph) the functional form of the velocity decay in these 3 cases is very different




(A) A constant frictional force ($b$) of the sort encountered when the object is sliding over a surface. The constant $b$, has the units of N-Newton's. Reformulating the constant $b$ has the units of acceleration (m/s$^2$). This is the familiar case of sliding friction on a surface where the friction is equal to coefficient of kinetic friction ($\mu_k$) times the normal force (N). In the case of an object on a flat horizontal plane N = $mg$ So:

$$b = \mu_k N$$

$$b = \mu_k g$$

The general solution of this constant frictional force problem goes as follows.

$$F = -b$$

$$ma = -ba = -b/m$$

$$\frac{dv}{dt} = -\frac{b}{m} = -b$$

$$dv = -bdt$$

$$\int_{v_0}^{v} dv = -\int_0^t bdt$$

$$v - v_0 = -bt$$

$$v = v_0 - bt$$

$$\frac{dx}{dt} = v_0 - bt$$

$$x = v_0 t - \frac{b}{2}t^2$$

(B) A frictional drag force which varies linearly with the velocity. The constant $b_1$, has the units of Ns/m or Kg/s. Using Newton's Law this can be reformulated with the constant $b_1$ has the units of 1/s. It is sometimes useful to parameterize the friction with in a way that takes the units into account by $b_1 = \frac{1}{\tau}$.

Here $\tau$ is a decay time constant and has the units of sec

The general solution to this problem goes as follows.

$$F = -b_1 v$$

$$ma = -b_1 v$$

$$a = -\frac{b_1}{m}v = -b_1 v$$

$$\frac{dv}{dt} = -\frac{b_1}{m} \qquad v = -\frac{v}{\tau}$$

$$b_1 = \frac{1}{\tau}$$

$$\frac{dv}{v} = -b_1 dt$$

$$\int_{v_0}^{v} dv/v = -\int_0^t b_1 dt$$

$$\int(v) = \int(v_0) = -bt$$

$$\left(\frac{v}{v_0}\right) = -b_1 t$$

$$\frac{v}{v_0} = e^{-b_1 t}$$

(C) A frictional drag force which varies quadratically with the velocity (as is often the case for higher velocity objects traveling in air). Here, the constant, $b_2$, has the units of Kg/m. Reformulating, one has $b_2$, with the units of 1/m and the decay length L has the units of $m$. For an object moving through a medium (air) $\rho$ is the medium density (in Kg/. m$^3$), C is the drag coefficient and A is the cross-section of the object perpendicular to the motion.

The general solution to this problem is as follows:

$$F = -b_2 v^2$$

$$ma = -b_2 v^2$$

$$a = -\frac{b_2}{m}v^2 = -b_2 v^2 = -\frac{v^2}{L}$$

$$\frac{dv}{dt} = -\frac{b_2}{m}v^2 = -b_2 v^2$$

$$\frac{dv}{v^2} = -b_2 dt$$

$$\int_{v_0}^{v} dv/v^2 = -\int_0^t b_2 dt$$

$$\frac{1}{v} - \frac{1}{v_0} = -b_2 t$$

$$\frac{v}{v_0} = \frac{1}{1+\frac{t}{\tau}}$$

$$\frac{1}{v_0}\frac{dx}{dt} = \frac{1}{1+v_0 b_2 t}$$

$$\int_{v_0}^{x} dx = -\int_0^t \frac{1}{1+v_0 b_2 t} dt$$

$$x = \frac{1}{b_2}\int[][1+ve^{v b_2}t]|$$

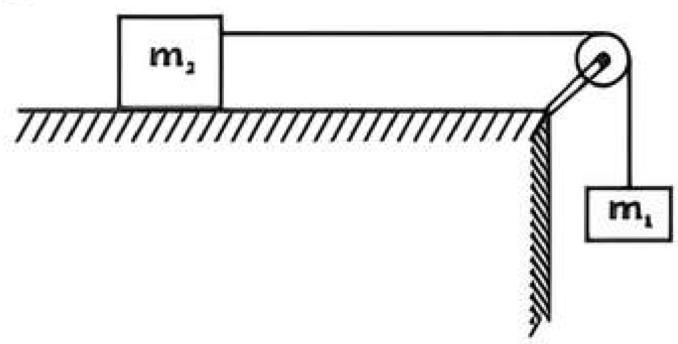
$$\frac{v}{v_0} = e^{-b_2}$$

**78.** For the system shown in the figure, the coefficient of static and kinetic friction between the block $m_2$ and the horizontal surface is $\mu_s$ and $\mu_k$.

If $m_2$ = 10 kg, $\mu_s$ = 0.4 and $\mu_k$ = 0.35

(A) Find the maximum value of $m_1$ so that the block $m_2$ does not move.

(B) If $m_1$ = 5 kg, find the acceleration of the system.




[Diksha](5m)

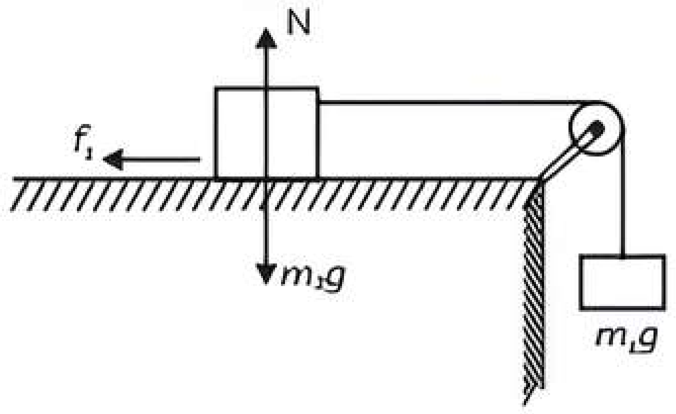
**Ans.** (A) The force which tries to accelerate the system is $(m_1 g)_{max}$, while the opposing force is $f_s$. For maximum value of $m_1$ under equilibrium condition.

$$(m_1 g)_{max} = (f_s)_{max} = \mu_s m_2 g$$

or $$(m_1)_{max} = \mu_s m_2$$

Here, $\mu_s = 0.4$, $m_2 = 10$ kg.

Thus, $$(m_1)_{max} = (0.4)(10) = 4 \text{ kg.}$$




(B) When $m_1 = 5$ kg, the system starts accelerating, $m_1$ accelerates downward and $m_2$ accelerates rightward. Let $a$ be the acceleration of the system, then

$$a = \frac{F_{net}}{M_{system}} = \frac{m_1 g - f_k}{m_1 + m_2}$$

or $a \left[\frac{m_1 - \mu k m_2}{m_1 + m_2}\right] g$ putting $m_1 = 5$ kg; $m_2 = 10$ kg; $\mu_k = 0.35$; $g = 10$ m/s$^2$.

We get, $a = \left[\frac{5-(0.35)(10)}{5+10}(10)\right] = 1 \text{ m/s}^2$